Mathematics 3201
Unit 2: Counting Methods
Review Questions

1. (A) How many ways can the letters in the word MISSISSIPPI be arranged?

$\frac{11!}{4!4!2!} = 34650$

(B) How many arrangements are there if the first letter has to be S?

$\frac{10!}{3!4!2!} = 12600$

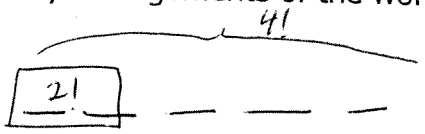
2. How many 4-digit numbers can be formed using the digits 1-9 if repetition is allowed?

$9^4 = 6561$

3. There are six coloured marbles in a box and you take one out at a time. How many different ways can you take out four marbles?

${}_6P_4 = 360$

4. (A) How many arrangements of the word "PHONE" exist if the letters O and E have to be together?



$2!\,4! = 48$

(B) How many arrangements are there if the letters O and E can't be together?

All − when they're together
$5! - 48 = 72$

5. How many passwords using at least 3 but no more than 5 digits can be made using the digits of 46723819 where repetition is not allowed?

$8\,7\,6 + 8\,7\,6\,5 + 8\,7\,6\,5\,4$

$336 + 1680 + 6720 = 8736$

6. How many ways can you order the letters from the word TREES if:

A) S must be at the beginning

$\underline{S}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ /2!$ (repetition of E) $= 12$

B) the R must be in the middle

$(\underline{4}\ \underline{3}\ \underline{R}\ \underline{2}\ \underline{1})/2! = 12$

C) It begins with exactly one E

can't be E!

$\underline{E}\ \underline{3}\ \underline{3}\ \underline{2}\ \underline{1} = 18$

D) Consonants and vowels alternate

$(\underline{3}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1})/(2!) = 6$

7. How many ways can 4 rock, 5 pop, and 6 classical albums be ordered if all the albums of the same genre must be kept together?

$${}_4P_4 \times {}_5P_5 \times {}_6P_6 = 24 \times 120 \times 720 = 2{,}073{,}600$$

8. How many ways can you order the letters in FORTUNES if the vowels must never be together?

All - together
8! - 4320
= 36,000

together
3! _ _ _ _ _ _ _ = 3!6!
6!
= 4320

9. If an ice cream dessert can have 2 toppings, and 9 are available, how many different topping selections can you make?

$${}_9C_2 = 36$$

10. There are 9 possible toppings for a sandwich, but you only want 4 toppings, one of which must be pickles. How many different sandwiches can be made?

$${}_1C_1 \times {}_8C_3 = 56$$

11. If a crate of radio controlled cars contain 10 working cars and 4 defective cars, how many ways can you take out 5 cars and have only three work?

$${}_{10}C_3 \times {}_4C_2 = 120 \times 6 = 720$$

12. If a student must select two courses from Group A (Math 3201, Chem 3202, Physics 3204 and Biology 3201), two courses from Group B(Eng 3201, Science 3200) and one course from Group C (Math 3208, Earth Systems 3208, French 3201), how many combinations are there?

$${}_4C_2 \times {}_2C_2 \times {}_3C_1 = 6 \times 1 \times 3 = 18$$

13. A student council of 5 members is to be formed from a selection pool of 6 boys and 8 girls. How many councils can have

A) Jason on the council

$${}_{13}C_4 = 715$$

B) Katie, but not John

$${}_1C_1 \times {}_{12}C_4 = 495$$

C) Zach, but not Caroline, Allison or Mark

$${}_1C_1 \times {}_{11}C_4 = 330$$

D) At least 3 boys, but one of these boys can't be Brian

$${}_5C_3 \times {}_8C_2 + {}_5C_4 \times {}_8C_1 + {}_5C_5 \times {}_8C_0$$
$$(10)(28) + (5)(8) + (1)(1) = 321$$

14. A research team of 7 people is to be formed from 8 chemists, 4 politicians, 3 economists and 2 biologists. How many teams have:

A) At least 5 chemists?

$\binom{8}{3}\binom{9}{2} + \binom{8}{6}\binom{9}{1} + \binom{8}{7} = 56(36) + 28(9) + 8 = 2276$

B) Exactly three economists

$\binom{3}{3}\binom{14}{4} = (1)(1001) = 1001$

C) Four chemists, but no economists

$\binom{8}{4}\binom{6}{3} = (70)(20) = 1400$

D) At least two biologists

$\binom{17}{7} - \binom{15}{7} - \binom{2}{1}\binom{15}{6} =$

$19448 - 6435 - 10010 = 3003$

15. It there are 8 boys & 7 girls in a selection pool and a school council of President, VP, treasurer and Secretary to be formed, in how many ways can

A) exactly one boy be on council

$8 \quad 7 \quad 6 \quad 5 = 1680$

B) exactly two girls be on council

$7 \quad 6 \quad 8 \quad 7 = 2352$

C) no boys on council

${}_7P_4 = 840$

16. If a sports team with six unique positions is to be formed from 5 men and 7 women, in how many ways can two positions be filled by men and four positions by women?

${}_5P_2 \times {}_7P_4 = 20 \times 840 = 16800$

17. Simplify:

A) $\frac{(n-1)!}{(n-3)!} = (n-1)(n-2)$

$n^2 - 3n + 2$

B) $\frac{(3n+2)!}{(3n+3)!} = \frac{1}{3n+3}$

18. Solve:

A) $\frac{(n-1)!}{(n-3)!} = 2$

$n = 3$

B) $(n+2)! = 12n!$

$n = 2$

19. Solve:

(A) ${}_nC_2 = 15$

$n = 6$

(B) ${}_{n+1}C_n = 20$

$n = 19$

(C) ${}_{n+1}P_2 = 20$

$n = 4$