

Section 6.4: Solving Trigonometric Equations Using Identities

**Example 1:** Solve each equation algebraically over the domain  $0 \leq x \leq 2\pi$

a)  $\cos^2 x + 1 - \cos x = 0$

rewrite

$$2\cos^2 x - 1 + 1 - \cos x = 0$$

$$2\cos^2 x - \cos x = 0$$

$$\cos x (2\cos x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

b)  $1 - \cos^2 x = 3\sin x - 2$

$$\sin^2 x = 3\sin x - 2$$

$$\sin^2 x - 3\sin x + 2 = 0$$

$$(\sin x - 2)(\sin x - 1) = 0$$
~~$$\sin x = 2 \quad \text{or} \quad \sin x = 1$$~~

No solution  $x = \pi/2$

**Example 2:** Solve the equation  $\cos^2 x = \cot x \sin x$  over the domain  $0 \leq x \leq 360^\circ$

$$\cos^2 x = \frac{\cos x \cdot \cancel{\sin x}}{\cancel{\sin x}}$$

$$\cos^2 x = \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad \cos x - 1 = 0$$

$$\begin{array}{l} x = 90^\circ, 270^\circ \\ \cos x = 1 \\ x = 0^\circ, 360^\circ \end{array}$$

**Example 3:** Solve algebraically  $\sin 2x = \sqrt{2} \cos x$ . (General solution!)

$$2 \sin x \cos x = \sqrt{2} \cos x$$

$$2 \sin x \cos x - \sqrt{2} \cos x = 0$$

$$\cos x (2 \sin x - \sqrt{2}) = 0$$

$$\cos x = 0 \quad \text{or} \quad 2 \sin x - \sqrt{2} = 0$$

$$x = 90^\circ, 270^\circ$$

$$x = 90^\circ + 180k, k \in \mathbb{I}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$x = 45^\circ \quad x = 135^\circ$$

$$x = 45^\circ + 360k, k \in \mathbb{I}$$

$$135^\circ + 360k, k \in \mathbb{I}$$

**Example 4:** Solve algebraically  $2\sin x = 7 - 3\csc x$