

M3200 Extra Practice 6.2

1. Calculate the exact value of each expression.

a) $\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ$

b) $\cos \frac{5\pi}{12} \cos \frac{\pi}{12} - \sin \frac{5\pi}{12} \sin \frac{\pi}{12}$

c) $\cos 75^\circ$

d) $\sin \frac{11\pi}{12}$

e) $\frac{\tan \frac{2\pi}{3} + \tan \frac{\pi}{12}}{1 - \tan \frac{2\pi}{3} \tan \frac{\pi}{12}}$

f) $\cot 165^\circ$

g) $\cos(-165^\circ)$

2. Simplify completely.

a) $\cos 2x \cos x + \sin 2x \sin x$

b) $\sin(30^\circ + x) + \sin(30^\circ - x)$

c) $\cos x \cos y (\tan x + \tan y)$

d) $2 \cos^2 10^\circ - 1$

e) $2 \sin 35^\circ \cos 35^\circ$

f) $\frac{2 \tan 25^\circ}{1 - \tan^2 25^\circ}$

3. a) Prove $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$

b) Prove $\sin(\pi + x) = -\sin x$

Practice 6.2

$$\begin{aligned} & \sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ \\ &= \sin(75^\circ + 15^\circ) \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{b) } & \cos \frac{5\pi}{12} \cos \frac{\pi}{12} - \sin \frac{5\pi}{12} \sin \frac{\pi}{12} \\ &= \cos \left[\frac{5\pi}{12} + \frac{\pi}{12} \right] \\ &= \cos \left[\frac{6\pi}{12} \right] \\ &= \cos \left[\frac{\pi}{2} \right] = 0 \end{aligned}$$

$$\begin{aligned} \text{c) } \cos 75^\circ &= \cos [45^\circ + 30^\circ] \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{d) } \sin \frac{11\pi}{12} &= \sin \left[\frac{3\pi}{4} + \frac{\pi}{6} \right] \\ &= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \cos \frac{3\pi}{4} \sin \frac{\pi}{6} \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2} \right) \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{(e) } & \frac{\tan \frac{2\pi}{3} + \tan \frac{\pi}{12}}{1 - \tan \frac{2\pi}{3} \tan \frac{\pi}{12}} \\ &= \tan \left(\frac{2\pi}{3} + \frac{\pi}{12} \right) \\ &= \tan \frac{3\pi}{4} \end{aligned}$$

$$f) \cot 165^\circ = \frac{1}{\tan 165^\circ}$$

$$= \frac{1}{\tan [135^\circ + 30^\circ]}$$

$$= \frac{1}{\tan 135^\circ + \tan 30^\circ}$$

$$= \frac{1 - \tan 135^\circ \tan 30^\circ}{1 + \tan 135^\circ \tan 30^\circ}$$

$$= \frac{1 - (-1) \left(\frac{\sqrt{3}}{3}\right)}{1 + (-1) \left(\frac{\sqrt{3}}{3}\right)}$$

$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}}$$

$$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \quad \checkmark$$

-3.73

$$g) \cos(-165^\circ) = \cos(195^\circ) = \cos(135^\circ + 60^\circ)$$

$$= \cos 135^\circ \cos 60^\circ - \sin 135^\circ \sin 60^\circ$$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= -\frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\# 2a) \cos 2x \cos x + \sin 2x \sin x$$

$$= \cos 2x \cos x + 2 \sin x \cos x \sin x$$

$$= \cos 2x \cos x + 2 \sin^2 x \cos x$$

$$= (1 - 2 \sin^2 x) \cos x + 2 \sin^2 x \cos x$$

$$= \cos x - 2 \sin^2 x \cos x + 2 \sin^2 x \cos x$$

$$= \cos x$$

$$\# 2b) \sin(30^\circ + x) + \sin(30^\circ - x)$$

$$= \sin 30^\circ \cos x + \cos 30^\circ \sin x + \sin 30^\circ \cos x - \cos 30^\circ \sin x$$

$$= 2 \sin 30^\circ \cos x$$

$$= 2 \cdot \frac{1}{2} \cos x$$

$$= \cos x$$