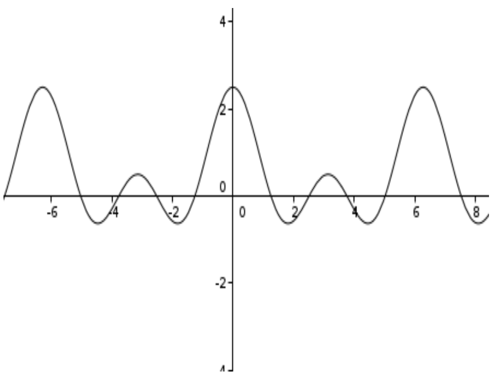
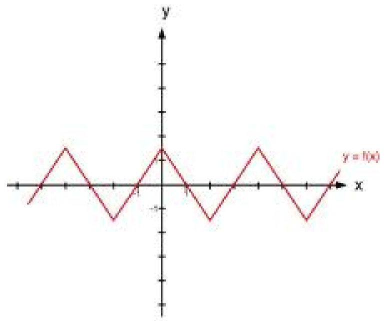
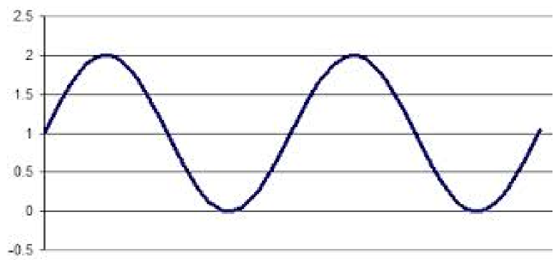
Chapter 5: Trigonometric Functions and Graphs
Section 5.1: Graphing Sine and Cosine Functions

Periodic Function: a function for which the dependent variable takes on the same set of values over and over again as the independent variable changes



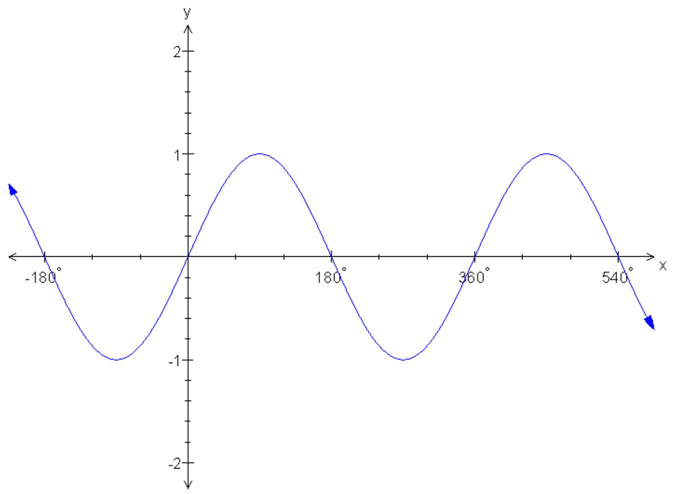


Sinusoidal Function: a periodic function that looks like waves



Important Terms for Sinusoidal Functions

period: **the change in the x-corresponding to the cycle of the function. A cycle is a portion of the graph from one point to the point at which the graph starts to repeat**

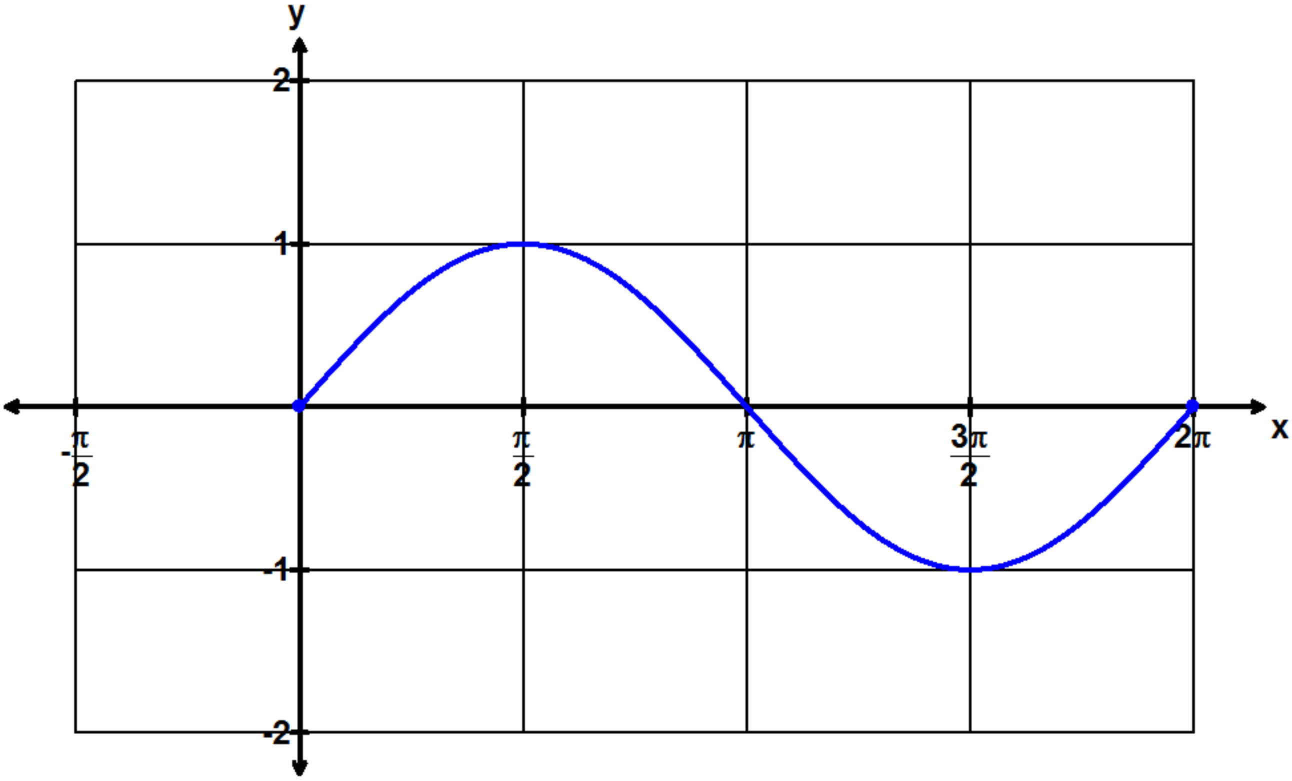


amplitude, sinusoidal axis, local max/local min

Graph of $y = \sin x$

There's 5 key points we associate with the graph, since these points will help us determine the characteristics of the graph (amplitude, domain, range, period, range and zeros, sinusoidal axis). However we are not limited to these points.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	-1	0



Amplitude

y-intercept:

period

x-intercepts:

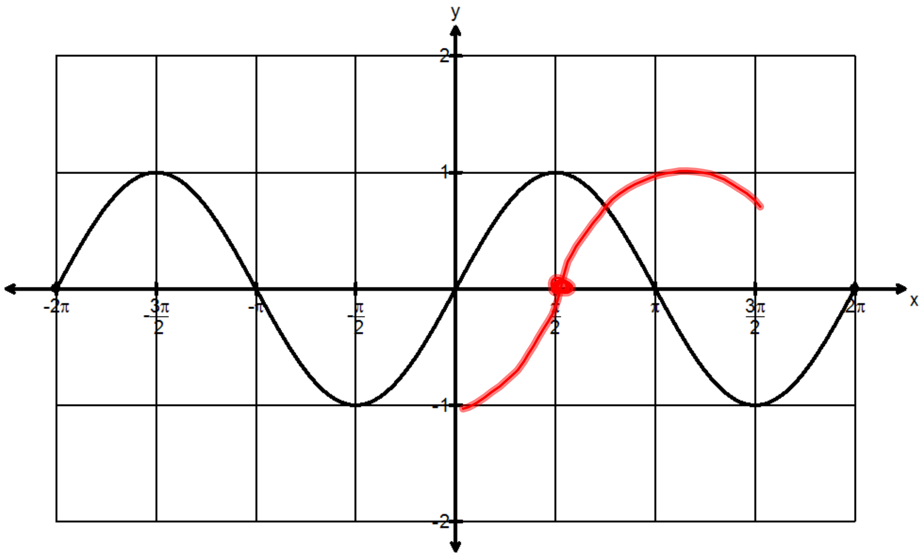
Domain

max value:

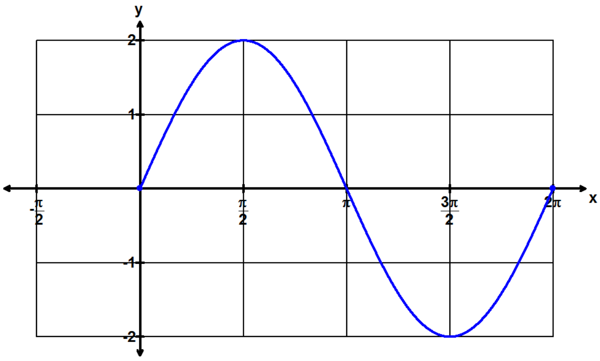
range

min value:

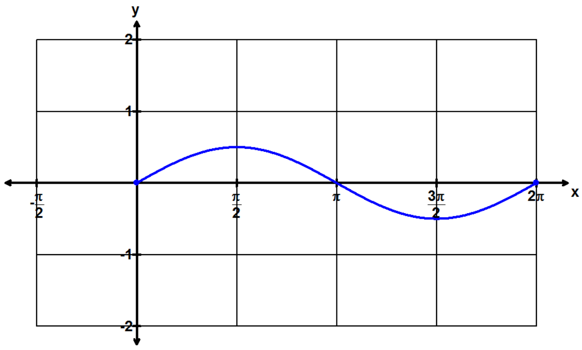
y = sin x



Determining the Amplitude of a Sine Function



y = 2sinx

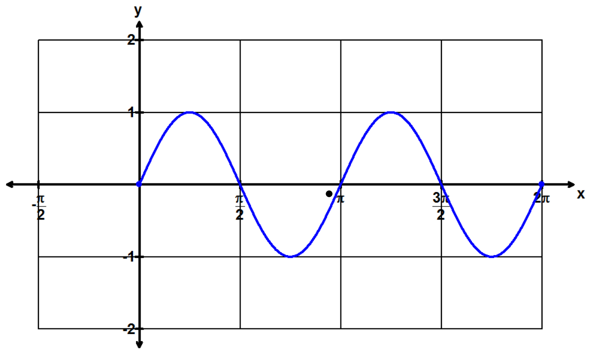


y= 0.5sinx

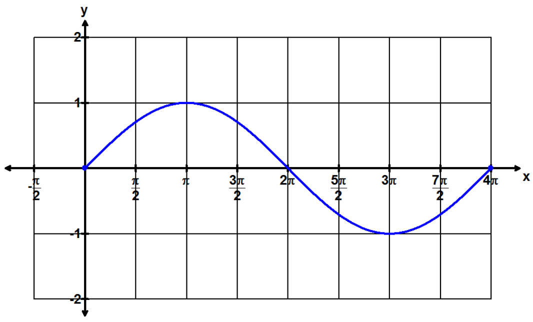
- Any function of the form y = af(x) is related to y = f(x) by a vertical stretch factor of $|a|$

- for the function y = asinx, the amplitude is $|a|$

Determining the Period of a Transformed Sine Function



y= sin 2x



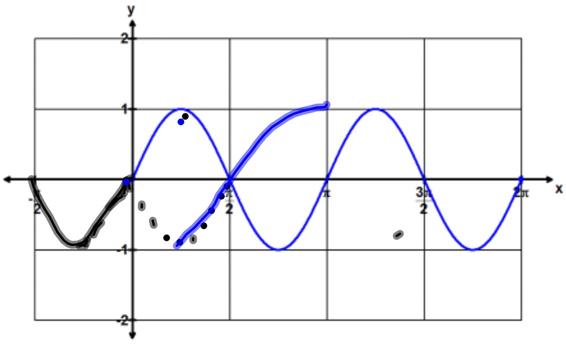
y=sin 0.5x

A function of the form y = f(bx) is related to y = f(x) by a horizontal stretch by a factor of $\frac{1}{|b|}$ about the y-axis.

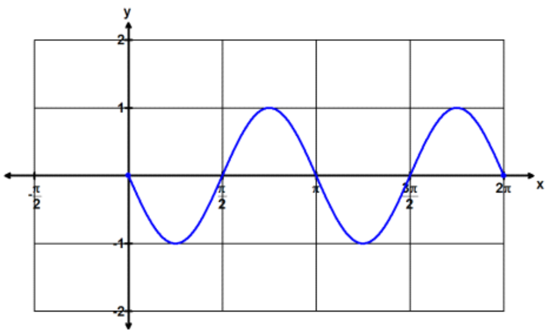
Changing the value of b affects the period of a sinusoidal function.

$$NewPeriod = \frac{2\pi}{|b|}$$

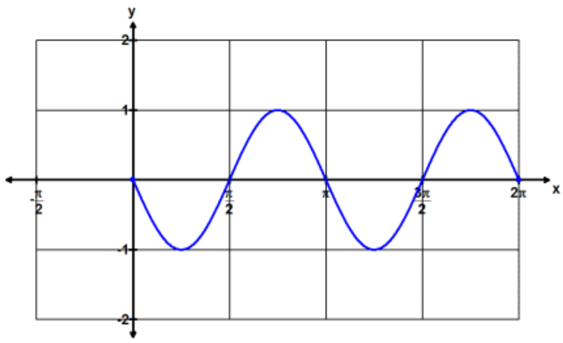
Reflections



y = sin2x



y = -sin2x



y = sin (-2x)

Example 1: Sketch the graph of y = -3sin2x for at least one cycle. Determine the amplitude, period, max and min values, x-intercepts, y- intercepts, domain and range and equation of sinusoidal axis.

OLD

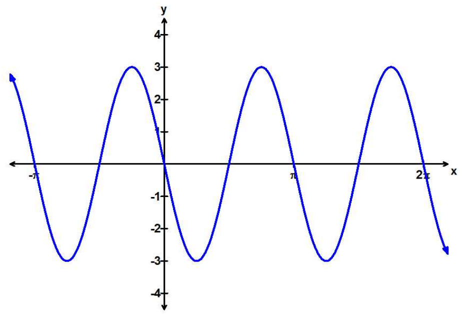
$y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

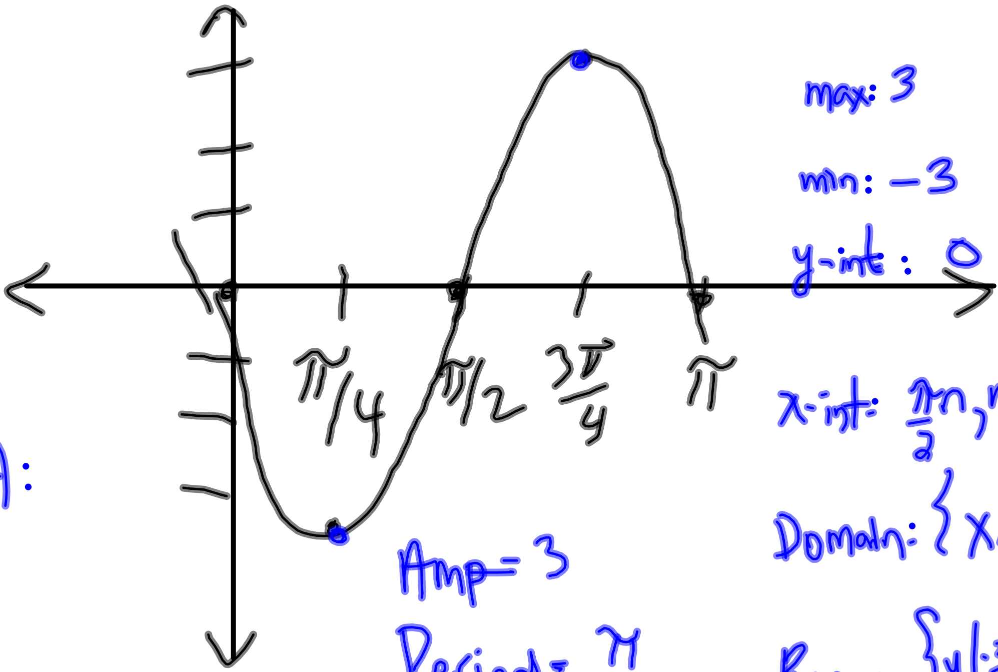
NEW

$y = -3\sin 2x$

$\frac{1}{2}x$	$-3y$
0	0
$\pi/4$	-3
$\pi/2$	0
$3\pi/4$	+3
π	0



SA:



max: 3

min: −3

y-int: 0

x-int: $\frac{\pi}{2}n, n \in I$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid -3 \leq y \leq 3, y \in R\}$

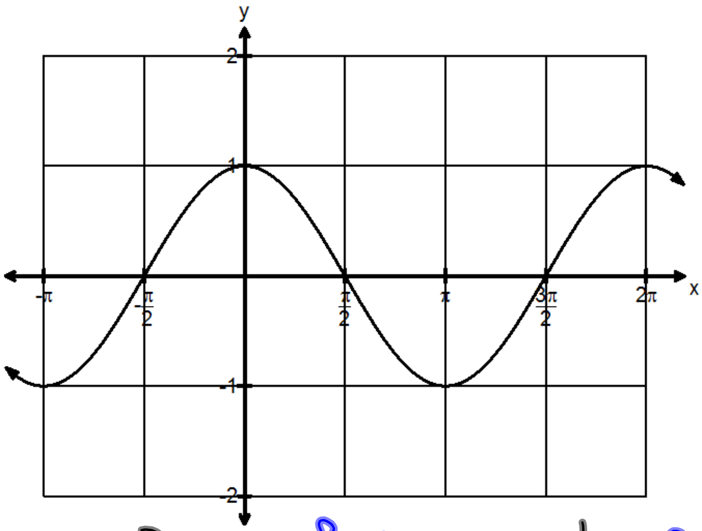
Amp = 3

Period = π

Pg 233 #4ac, #5 a,c, #7, 9a,c, #11b,c,d,

Graph of *y* = cos *x*

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	0	-1	0	1



Amp: 1

Period: 2π

Domain: $\{x | x \in R\}$ x-int: $\frac{\pi}{2} + \pi n, n \in I$

Range: $\{y | -1 \le y \le 1, y \in R\}$ y-int: 1

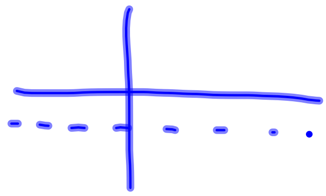
Sinusoidal axis: $y = 0$

#4b,d,5b,d,8,9bd,11a

Section 5.2: Transformations

Example 1: Sketch the graph of the function

$$y = -2\cos(x + \pi) - 1$$



State the period, amplitude, sinusoidal axis, domain and range.

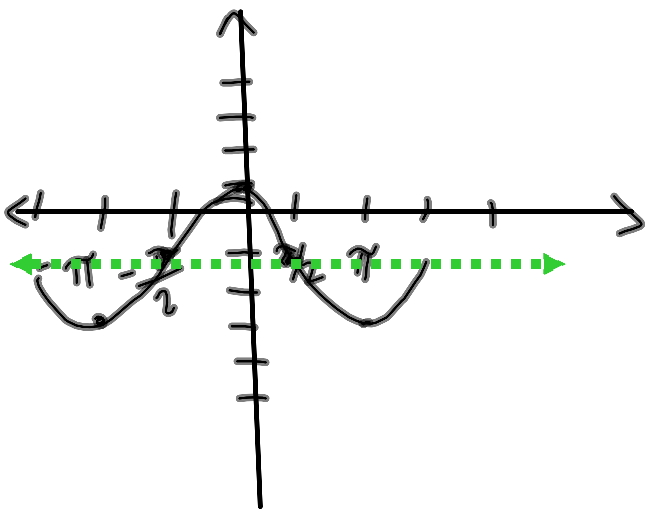
$(x, y) \rightarrow (x - \pi, -2y - 1)$

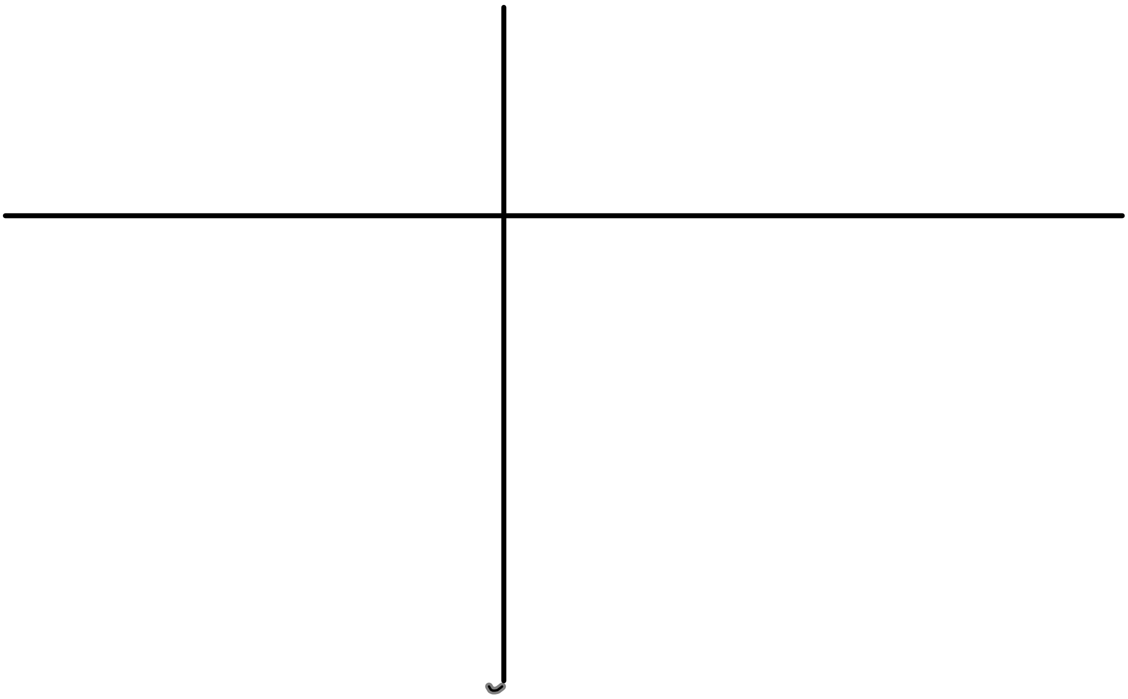
OLD

x	y
0	1
$\pi/2$	0
π	−1
$3\pi/2$	0
2π	1

NEW

$x-\pi$	$-2y-1$
$-\pi$	−3
$-\pi/2$	−1
0	1
$\pi/2$	−1
π	−3





Example 2: Sketch the graph of the function

$$y = 3\sin(2x - \frac{2\pi}{3}) + 2$$

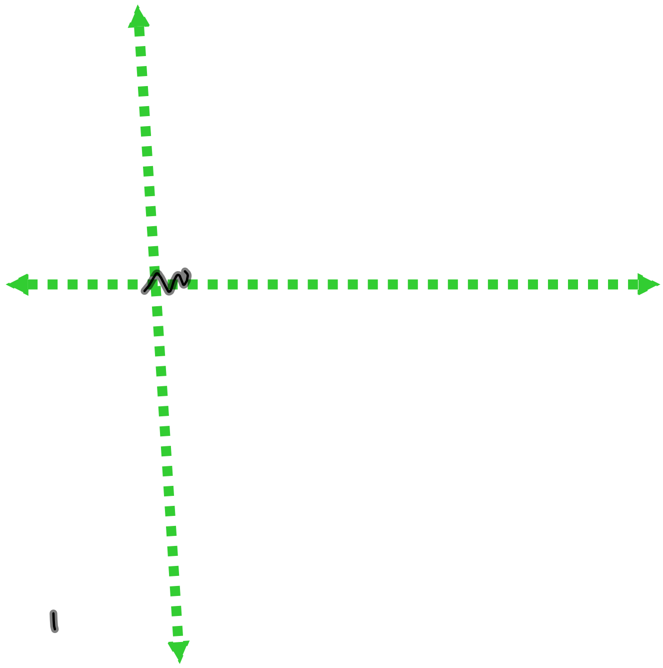
State the period, amplitude, sinusoidal axis, domain and range.

$$y = 3\sin 2(x - \frac{\pi}{3}) + 2$$

$$(x, y) \to (\frac{1}{2}x + \frac{\pi}{3}, 3y + 2)$$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	−1
2π	0

$\frac{1}{2}x + \frac{\pi}{3}$	$3y + 2$
$\pi/3$	2
$7\pi/12$	5
$5\pi/6$	2
$13\pi/12$	−1
$4\pi/3$	2



Example 3: Given $\frac{1}{3}(y+1)=\cos 2(x-30^0)$

State the amplitude, sinusoidal axis (horizontal central axis), vertical displacement, local max/min, period, phase shift, domain and range

$y=3\cos 2(x-30^\circ)-1$

Amp: 3

SA: $y=-1$

Vertical displacement: -1

Max: $-1+3=2$

min: $-1-3=-4$

period: $360^\circ\left(\frac{1}{2}\right)$

$=180^\circ$

phase shift $=30^\circ$

Domain: $x\in R$

Range: $-4\leq y\leq 2$

Pg 250 #1a,c,e,f,
#2(don't graph)a,c,f
#3a,
#4
#5
#14

Determining the Equation from a Graph

y = □ (sin □ (x - □) + □

amp. HS/period phase shift SA

("middle of upward slope")

y = □ (cos □ (x - □) + □

("top of the bump")

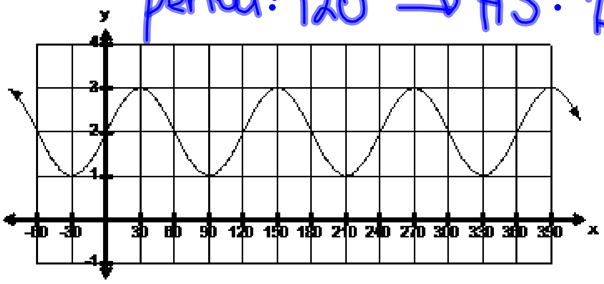
Example 1: The graphs below shows the function y = f(x).

A) Write the equation in the form y = a sin b(x-c) + d, a > 0

B) Write the equation in the form y = a cos b(x-c) + d, a > 0

amp: 1 SA: $y=2$

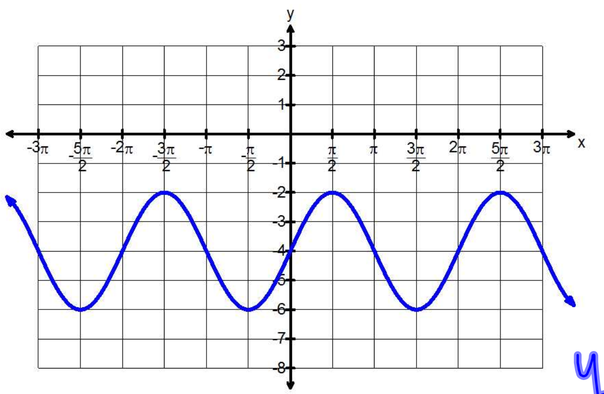
period: $120° \Rightarrow$ HS: $1/3$



$y = 1\sin 3x + 2$

$y = \sin 3x + 2$

$y = \cos 3(x - 30°) + 2$



$y = 2\sin x - 4$

$y = 2\cos(x - \pi/2) - 4$

amp. 2

SA: $y=-4$

period: $2\pi \Rightarrow$ HS: 1

Pg 250 #5, 6ac, 7, 14, 15, 16

Contextual Problems

Example 1:

Prince Rupert, British Columbia, has the deepest natural harbour in North America. The depth, d, in metres, of the berths for the ships can be approximated by the equation $d(t) = 8 \cos \frac{\pi}{6}t + 12$, where t is the time, in hours, after the first high tide.

a) Graph the function for two cycles.
b) What is the period of the tide?
c) An ocean liner requires a minimum of 13 m of water to dock safely. From the graph, determine the number of hours per cycle the ocean liner can safely dock.
d) If the minimum depth of the berth occurs at 6 h, determine the depth of the water. At what other times is the water level at a minimum? Explain your solution.

pg 248 your turn, 10a, 23,24