Section 3.5: Conditional Probability

Dependent Events: events whose outcomes are affected by one another.

Example (1):

Cards are drawn from a standard deck of 52 cards (without replacement). Calculate the probability of obtaining:

(a) a king, then another king

$$\frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

(b) a club, then a heart

$$\frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$$

(c) a black card, then a heart, then a diamond

$$\frac{26}{52} \times \frac{13}{51} \times \frac{13}{50} = \frac{169}{5100}$$

Conditional Probability:

- the probability of an event occurring given that another event has already occurred.
- The events are dependent!

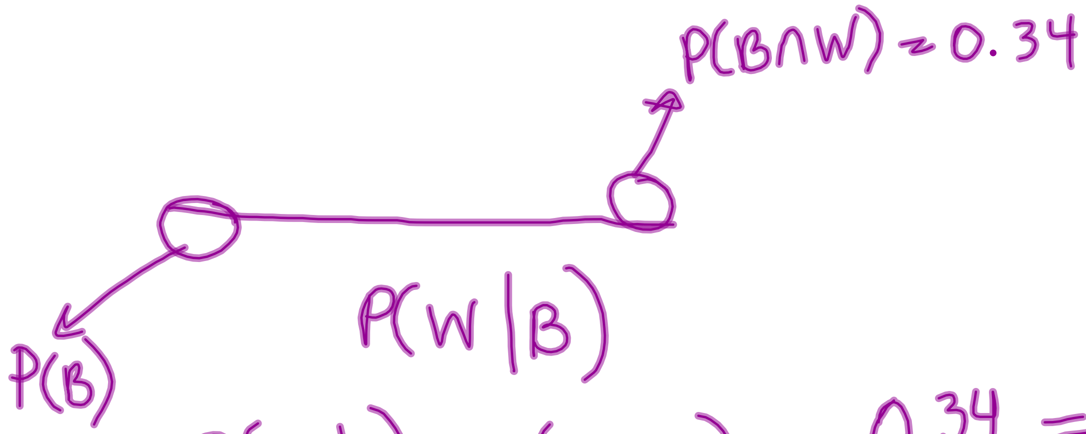
If event B depends on event A occurring, then the probability that both events will occur can be determined using the formula:

$P(A \cap B) = P(A) \times P(B|A)$ where $P(B|A)$ is read as "the probability event B will occur given that A has already occurred"

A & B

This formula can be arranged as $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

when we are trying to determine the probability of B after A has occurred.



$$P(W|B) = \frac{P(B \cap W)}{P(B)} = \frac{0.34}{0.47} = 0.72$$

Example 3:

$P(1 \cap 2) = 0.25$

A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

$P(1) = 0.42$

$P(2|1) = ?$

$$P(2|1) = \frac{P(1 \cap 2)}{P(1)} = \frac{0.25}{0.42} = 0.60$$

$\boxed{60\%}$

Example 4:

At Vikings School, 40% of the boys play soccer and 20% percent of those that play soccer also play football. What percent of boys play soccer and football.

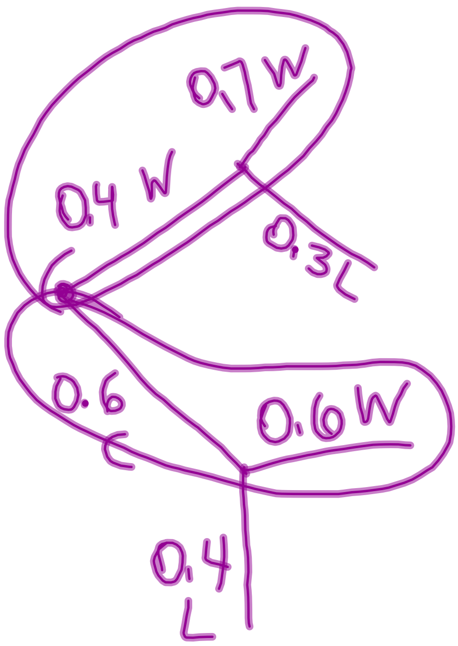
$P(S \cap F)$

$P(S) = 40\% = 0.4$

$P(F|S) = 0.2$

$$P(S \cap F) = (0.4)(0.2) = 0.08$$

$\boxed{8\%}$



$$P(\text{winning}) = (0.4)(0.7) + (0.6)(0.6)$$
$$= 0.64$$