

Key

Math 3201 – Section 3.3 (Probabilities using Counting Methods)

Investigate the Math (p. 151 in text)

As a volunteer activity, 10 students want to put on a talent show at a retirement home. To organize the show, 3 of these students will be chosen at random to form a committee. Victoria really wants to be on this committee, since her grandmother lives at the home. Each student's name will be written on a slip of paper and placed in a hat. Then 3 names will be drawn.

A. Is Victoria's name just as likely to be drawn as any other name? Explain.

Yes. Each slip has an equal chance of being chosen as there is only one of each submitted.

B. Does the order in which the names are drawn matter? Explain.

No - committee

C. In how many different ways can 3 names be drawn from a hat with 10 names? Explain.

$${}_{10}C_3 = 120$$

D. In how many different ways can Victoria's name be drawn with 2 other names? Explain.

Victoria ← $1 \times {}_9C_2 = 1 \times 36 = 36$

E. What is the probability that Victoria's name will be drawn? Explain.

$$P(\text{Victoria Chosen}) = \frac{n(\text{Victoria and 2 other names})}{n(3 \text{ names})} = \frac{36}{120} = 0.3$$

Example 1:

Two cards are picked without replacement from a deck of 52 playing cards. What is the probability that both are kings? 30% chance.

$$P(2 \text{ kings}) = \frac{n(2 \text{ kings})}{n(\text{any 2 cards})} = \frac{{}_4C_2}{{}_{52}C_2} = \frac{6}{1326} = \frac{1}{221} = 0.0045$$

0.45%

Example 2:

The student council forms a sub-committee of five council members to look at how funds raised should be spent. If there are a total of 10 student council members, 4 males and 6 females, what is the probability that the sub-committee will consist of exactly 3 females?

$$P(\text{exactly 3 females}) = \frac{n(3 \text{ females})}{n(\text{all possible})}$$

$$= \frac{{}^6C_3 \times {}^4C_2}{{}^{10}C_5} = \frac{20 \times 6}{252} = \frac{120}{252} = \frac{10}{21} = 0.48$$

48%

Example 3:

A bag of marbles contains 5 red, 3 green, and 6 blue marbles. If a child grabs three marbles from the bag, determine the probability that:

A) at least one is blue?

$$P(\text{at least one blue}) = \frac{n[\text{All - no blue}]}{n(\text{all possible})} = \frac{{}^{14}C_3 - {}^8C_3}{{}^{14}C_3} = \frac{364 - 56}{364} = \frac{308}{364} = \frac{11}{13} = 0.846$$

B) the first is red, the second is green, and the third is blue?

$$P(R, G, B) = \frac{{}_5P_1 \times {}_3P_1 \times {}_6P_1}{{}_{14}P_3} = \frac{15}{364} \Rightarrow \text{Permutation!}$$

85%

C) one is red, one is green, and one is blue? *Combination!*

$$P(R, G, B) = \frac{{}_5C_1 \times {}_3C_1 \times {}_6C_1}{{}_{14}C_3} = \frac{5 \times 3 \times 6}{364} = \frac{45}{182} = 0.247$$

Practice Questions

1. There are 7 teachers and 3 administrators at a conference. Find the probability of three different door prizes being awarded to teachers only.

$$\frac{{}_7P_3}{{}_{10}P_3}$$

2. A letter is chosen at random from the word COMMUNICATE. What is the probability that

A) a consonant is drawn?

B) the letter is a C?

6 consonant
11 total

$$\frac{{}_6C_1}{{}_{11}C_1}$$

$$\frac{{}_2C_1}{{}_{11}C_1} = \frac{2}{11}$$

2 C's
11 total

3. A box contains eleven balls numbered from one to eleven inclusive. Two balls are drawn without replacement. What is the probability (as an exact fraction) that both balls show odd numbers?

$$P(\text{both odd}) = \frac{{}^6C_2}{{}^{11}C_2} = \frac{15}{55} = \frac{3}{11}$$

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑪

4. There are ten boys and seven girls in a class. If a group of five students are chosen at random to do an experiment, find the probability that:

A) exactly two are girls?

$$= \frac{{}^2C_2 \cdot {}^3C_3}{{}^{17}C_5} = \frac{1 \cdot 1}{6100}$$

B) there are no boys?

All Girls

$$= \frac{{}^7C_5}{{}^{17}C_5}$$

C) there are 4 boys?

$$\frac{{}^{10}C_4 \cdot {}^7C_1}{{}^{17}C_5}$$

D) at least 4 are boys?

$$\frac{{}^{10}C_4 \cdot {}^7C_1 + {}^{10}C_5}{{}^{17}C_5}$$

5. A marble is drawn at random from a box containing 10 red, 30 yellow, 20 blue and 10 pink marbles. Find, as an exact fraction, the probability that the marble drawn is:

A) yellow or red

$$\frac{30}{70} + \frac{10}{70}$$

B) not blue

$$\frac{50}{70}$$

C) green

$$P(\text{green}) = 0$$

D) red, pink, or blue

$$\frac{10}{70} + \frac{10}{70} + \frac{20}{70}$$