Math 3200 - Chapter Eleven
Permutations, Combinations, and the Binomial Theorem

1. Three identical door prizes are to be given to three lucky people in a crowd of 100. In how many ways can this be done? $_{100}C_3 = 161\ 700$

2. The license plates in a certain province consist of 3 letters followed by 3 non-zero digits. How many such license plates are possible? 26 26 26 9 9 9

3. How many numbers from 1000 and 9999, inclusive,:
a) contain no zeros 9 9 9 9 = 6561
b) contain no ones 8 9 9 9 = 5832
c) begin with an even digit and end with an odd digit 4 10 10 5 = 2000

4. A lock has a dial with 50 numbers on it . To open it you must turn left to a number, right to a number, and then left to a number. How many possibilities are there if:
a) the 3 numbers must be different 50 49 48 = 117 600
b) the numbers are not necessarily different 50 50 50 = 125 000

5. A student must take 4 final exams that are to be scheduled by the computer for the morning and afternoon testing periods on Monday through Friday. How many ways are there to schedule the 4 exams? $_{10}P_4 = 5040$

6. A railway has 30 stations. On each ticket, the departure station and the destination station are to be printed.
a) How many different types of tickets are there? $_{30}P_2 = 870$
b) If a ticket could be used in either direction between two stations, how many different types of tickets would need to be printed? $_{30}C_2 = 435$

7. How many 6 letter "words" could be formed using all 6 letters:
a) radish $6! = 720$
b) squash $\frac{6!}{2!} = 360$

8. There are 3 roads from town A to Town B, 5 roads from B to C, and 4 roads from C to D. a) How many ways are there to go from A to D via B and C? b) How many different round trips are there? a) $3 \cdot 5 \cdot 4 = 60$ b) $60 \cdot 60 =$ ~~360~~ ~~129600~~ 3600

9. In Morse Code, letters, digits, and punctuation marks are represented by a sequence of dots and dashes. Sequences can be from 1 unit to 5 units in length. How many such sequences are possible? 2 | 2 2 =4 | 2 2 2 =8 | 2 2 2 2 =16 | 2 2 2 2 2 =32 $\boxed{\text{Total } 62}$

10. A teacher must pick 3 high school students from a class of 30 to prepare and serve food at a picnic. How many selections are possible?

$_{30}C_3 = 4060$

11. A Town Council consists of 8 members including the Mayor.
a) How many different committees of four can be chosen from this council? ${}_8C_4 = 70$
b) How many of these committees will include the Mayor? ${}_7C_3 = 35$
c) How many will not include the Mayor? $70 - 35 = 35$
d) Verify that the answer to part a is the sum of the answers from parts b and c.

12. Repeat #11 if the council has 9 members including the Mayor. a) ${}_9C_4 =$ b) ${}_8C_3$ c) ${}_9C_4 - {}_8C_3$

★ 13. If you have a \$5 bill, a \$10 bill, and a \$20 bill, how many different sums of money can you make using one or more of these bills? ${}_3C_1 + {}_3C_2 + {}_3C_3 = 7$

14. A pizza shop pepperoni, mushrooms, sausages, onions, anchovies, and peppers as toppings for their regular plain pizza. How many different pizzas can be made?

$${}_6C_0 + {}_6C_1 + {}_6C_2 + {}_6C_3 + {}_6C_4 + {}_6C_5 + {}_6C_6 = 64$$

15. a) How many 4 letter "words" can be formed using the 8 letters of TRIANGLE? $\underline{8}\ \underline{7}\ \underline{6}\ \underline{5} = 1680$
b) How many of the "words" in part a have no vowels? $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2} = 120$
c) How many "words in part a have at least one vowel?
All − No vowels = 1680 − 120 = 1560

16. How many 5 digit numbers contain at least 1 three?
All − NO 3's $\underline{9}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} - \underline{8}\ \underline{9}\ \underline{9}\ \underline{9}$

17. a) 5 boys and 5 girls stand in a line. How many arrangements are possible if all of the boys stand in succession? = 37512
b) if the boys and girls stand alternately?

★ $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ 14400 x 6 = 86400

b) $\underline{5}\ \underline{5}\ \underline{4}\ \underline{4}\ \underline{3}\ \underline{3}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1}$

X 2

= 14 400 x 2

$\boxed{= 28,800}$

37. What is the 6th term of the Binomial Theorem expansion of $\left(2a^2 - \frac{b}{4}\right)^8$?

Answers:

1. A	2. B	3. A	4. A	5. A	6. B	7. C
8. D	9. C	10. B	11. C	12. C	13. C	14. C
15. C	16. B	17. C	18. B	19. C	20. A	21. B
22. D	23. D	24. B	25. D	26. D		

27. $5! = 120$

28. A) ${}_{26}P_4 \times {}_{10}P_3 = 258\ 336\ 000$ B) ${}_{22}P_3 \times {}_{10}P_3 = 6\ 652\ 800$

29. A) $7! = 5040$

B) think of the two red cars as one and the three blue cars as one $\Rightarrow$ so you need to arrange 4 things (${}_4P_4 = 4! = 24$) but then you must consider that there are ${}_3P_3 = 3! = 6$ ways of arranging the 3 blue cars and ${}_2P_2 = 2! = 2$ ways of arranging the red cars, so the final answer is ${}_4P_4 \times {}_3P_3 \times {}_2P_2 = 24 \times 6 \times 2 = 288$ (note: cars of the same color are not identical)

C) This would be the answer in A) – answer in B) : 5040-288 = 4752

30. A) ${}_{25}C_3 = 2300$ B) ${}_{25}P_3 = 13800$

31. A) $\frac{12!}{5!4!3!} = 27720$ B) $\frac{8!}{5!3!} = 56$

32. A) ${}_{20}C_6 = 38760$

B) no females would be ${}_{12}C_6 = 924$, so using answer from A), we find # of ways to choose 6 winners with at least 1 female winner = 38 760-924 = 37 836

C) 3 male winners = ${}_{12}C_3 \times {}_8C_3 = 220 \times 56 = 12320$

4 male winners = ${}_{12}C_4 \times {}_8C_2 = 495 \times 28 = 13860$

5 male winners = ${}_{12}C_5 \times {}_8C_1 = 792 \times 8 = 6336$

6 male winners = ${}_{12}C_6 \times {}_8C_0 = 924$

So, the total number of ways to award the tickets so that at least 3 males are winners = 12320+13860+6336+924=33 440

D) 0 female winners = ${}_8C_0 \times {}_{12}C_6 = 924$

1 female winner = ${}_8C_1 \times {}_{12}C_5 = 8 \times 792 = 6336$

2 female winners = ${}_8C_2 \times {}_{12}C_4 = 28 \times 495 = 13860$