Proving Trigonometric Identities

Prove the following:

1. $\dfrac{1}{\sin x \cos x} - \dfrac{\cos x}{\sin x} = \tan x$

2. $\dfrac{\cot x(1+\tan^2 x)}{\tan x} = \csc^2 x$

3. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

4. $\dfrac{1-3\cos x-4\cos^2 x}{\sin^2 x} = \dfrac{1-4\cos x}{1-\cos x}$

5. $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

6. $\dfrac{\cos x}{\sec x - 1} - \dfrac{\cos x}{\tan^2 x} = \cot^2 x$

7. $\csc x \cos^2 x + \sin x = \csc x$

8. $\sin x + \cot^2 x \sin x = \csc x$

9. $\cot x + \tan x = \csc x \sec x$

10. $\sec^2 x + \tan^2 x \sec^2 x = \sec^4 x$

11. $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} = \cot x$

12. $\dfrac{1-\sin^2 x}{1+\cot^2 x} = \sin^2 x \cos^2 x$

13. $\dfrac{\tan^2 x}{1+\tan^2 x} = \sin^2 x$

14. $\dfrac{\tan x + \sin x}{1+\cos x} = \tan x$

15. $\dfrac{\sec x + \tan x}{\dfrac{1}{1-\sin x}} = \cos x$

16. $\sec x + \csc x = \dfrac{1+\tan x}{\sin x}$

17. $\sec x + \dfrac{1}{\cot x} = \dfrac{1+\sin x}{\cos x}$

18. $(\cot A + \tan A)^2 = \csc^2 A \sec^2 A$

19. $\dfrac{\tan x}{1+\sec x} + \dfrac{1+\sec x}{\tan x} = 2\csc x$

20. $\dfrac{\sec^4 x - 1}{\tan^2 x} = \tan^2 x + 2$

21. $\dfrac{\cos x - \tan x}{\sin x \cos x} = \csc x - \sec^2 x$

Proving Trig. Identities

#1. $\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} = \tan x$

$= \frac{1 - \cos^2 x}{\sin x \cos x}$

$= \frac{\sin^2 x}{\sin x \cos x}$

$= \frac{\sin x}{\cos x}$

$\frac{\sin x}{\cos x}$

#2. $\frac{\cot x (1 + \tan^2 x)}{\tan x} \Big| \csc^2 x$

$= \frac{\cot x (\sec^2 x)}{\tan x}$

$= \frac{\frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}$

$= \frac{\frac{1}{\sin x \cos x}}{\frac{\sin x}{\cos x}}$

$= \frac{1}{\sin x \cos x} \cdot \frac{\cos x}{\sin x}$

$= \frac{1}{\sin^2 x}$

$= \csc^2 x$

$\csc^2 x$

#3. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$

$= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$

$= \frac{\sin^2 x (\sin^2 x)}{\cos^2 x}$

$= \tan^2 x \sin^2 x$

✓

#4. $\dfrac{1-3\cos x-4\cos^2 x}{\sin^2 x}$ | $\dfrac{1-4\cos x}{1-\cos x}$

$= \dfrac{(1-4\cos x)(1+\cos x)}{\sin^2 x}$

$= \dfrac{(1-4\cos x)(1+\cos x)}{1-\cos^2 x}$

$= \dfrac{(1-4\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)}$

$= \dfrac{1-4\cos x}{1-\cos x}$ ✓

$\dfrac{1-4\cos x}{1-\cos x}$ ✓

#5 $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

$= \dfrac{\sin x}{1+\cos x} \cdot \dfrac{1-\cos x}{1-\cos x}$

$= \dfrac{\sin x - \sin x \cos x}{1-\cos^2 x}$

$= \dfrac{\sin x(1-\cos x)}{\sin^2 x}$

$= \dfrac{1-\cos x}{\sin x}$

#6. $\dfrac{\cos x}{\sec x - 1} - \dfrac{\cos x}{\tan^2 x} = \cot^2 x$

$= \dfrac{\cos x(\sec x+1)}{\sec x - 1(\sec x+1)} - \dfrac{\cos x}{\tan^2 x}$

$= \dfrac{\cos x \sec x + \cos x}{\sec^2 x - 1} - \dfrac{\cos x}{\tan^2 x}$

$= \dfrac{\cos x \sec x + \cos x}{\tan^2 x} - \dfrac{\cos x}{\tan^2 x}$

$= \dfrac{\cos x \sec x}{\tan^2 x}$

$= \dfrac{\cos x \frac{1}{\cos x}}{\tan^2 x} = \dfrac{1}{\tan^2 x} = \cot^2 x$

#7. $\csc x \cos^2 x + \sin x = \csc x$

$\frac{\cos^2 x}{\sin x} + \sin x$ | $\frac{1}{\sin x}$ ✓

$\frac{\cos^2 x + \sin^2 x}{\sin x}$ ✓

$\frac{1}{\sin x}$

#8. $\sin x + \cot^2 x \sin x = \csc x$

$\sin x (1 + \cot^2 x)$ | $\frac{1}{\sin x}$ ✓

$\sin x (\csc^2 x)$

$\frac{\sin}{\sin^2}$ ✓

$\frac{1}{\sin x}$

#9. $\cot x + \tan x = \csc x \sec x$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ | $\frac{1}{\sin x \cos x}$ ✓

$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$ ✓

$= \frac{1}{\sin x \cos x}$

#10. $\sec^2 x + \tan^2 x \sec^2 x = \sec^4 x$

$\sec^2 x (1 + \tan^2 x)$ | ✓

$\sec^2 x (\sec^2 x)$

$\sec^4 x$ ✓

#11. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$

$\frac{\sec \cos x - \sin^2 x}{\sin x \cos x}$ | $\frac{\cos x}{\sin x}$ ✓

$\frac{1 - \sin^2 x}{\sin x \cos x}$

$\frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x}$ ✓

#12 $\frac{1 - \sin^2 x}{1 + \cot^2 x} = \sin^2 x \cos^2 x$ ✓

$= \frac{\cos^2 x}{\sec^2}$

$\cos^2 x \sin^2 x$

#13 $\frac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x$ ✓

$\frac{\tan^2 x}{\sec^2}$

$= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x$ ✓

#14. $\dfrac{\tan x + \sin x}{1+\cos x} \quad \Big| \quad \tan x$

$= \dfrac{\tan x + \sin x\left[1-\cos x\right]}{1+\cos x\left[1-\cos x\right]}$

$= \dfrac{\tan x - \tan x\cos x + \sin x - \sin x\cos x}{1-\cos^2 x}$

$= \dfrac{\frac{\sin x}{\cos x} - \sin x + \sin x - \sin x\cos x}{1-\cos^2 x}$

$= \dfrac{\frac{\sin - \sin x\cos^2 x}{\cos x}}{1-\cos^2 x}$

$= \dfrac{\frac{\sin x(1-\cos^2 x)}{\cos x}}{1-\cos^2 x}$

$= \dfrac{\sin x(1-\cos^2 x)}{\cos x} \cdot \dfrac{1}{1-\cos^2 x}$

$= \dfrac{\sin x}{\cos x} = \tan x$ ✓

#15. $\dfrac{\sec x + \tan x}{\frac{1}{1-\sin x}} \quad \Big| \quad \cos x$

$= (\sec x + \tan x)\cdot(1-\sin x)$

$= \sec x - \sec x\sin x + \tan x - \tan x\sin x$

$= \dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x} + \dfrac{\sin x}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{1-\sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x}{\cos x}$ ✓

$= \cos x$

#16. $\sec x + \csc x = \dfrac{1+\tan x}{\sin x}$

$= \dfrac{1}{\cos x} + \dfrac{1}{\sin x}$

$= \dfrac{\sin x + \cos x}{\cos x \sin x}$ ✓

$\dfrac{1+\frac{\sin x}{\cos x}}{\sin x}$

$= \dfrac{\frac{\cos x + \sin x}{\cos x}}{\sin x}$

$= \dfrac{\cos x + \sin x}{\cos x \sin x}$ ✓

#17. $\sec x + \frac{1}{\cot x} \stackrel{?}{=} \frac{1+\sin x}{\cos x}$

$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$\frac{1+\sin x}{\cos x}$ ✓

#18. $(\cot A + \tan A)^2 = \csc^2 A \sec^2 A$

$\cot^2 A + \tan^2 A$

$\cot^2 A + 2\cot A \tan A + \tan^2 A$

$\cot^2 A + 2 + \tan^2 A$

$\cot^2 A + 1 + 1 + \tan^2 A$

$\csc^2 x + \sec^2 x$

Mistake in question
should be $= \csc^2 x \sec^2 x$

#19. $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = 2\csc x$

$\frac{\tan^2 x + (1+\sec x)^2}{(1+\sec x)\tan x}$

$\frac{\sec^2 x + 2\sec x + \sec^2 x}{(1+\sec x)\tan x}$

$\frac{2\sec^2 x + 2\sec x}{(1+\sec x)\tan x}$

$\frac{2\sec x(\sec x + 1)}{(1+\sec x)\tan x}$

$\frac{2\sec x}{\tan x}$ $\frac{2}{\cos x}$ ✓

#20. $\frac{\sec^4 x - 1}{\tan^2 x} = \tan^2 x + 2$

$\frac{(\sec^2 x - 1)(\sec^2 x + 1)}{\tan^2 x}$ ✓

$\frac{(\tan^2 x)(\sec^2 x + 1)}{1}$

$\tan^2 x + \sec^2 x + 1$ ✓

$\tan^2 x + 2$

#21. $\dfrac{\cos x - \tan x}{\sin x \cos x} = \csc x - \sec^2 x$

$\dfrac{\cos x - \frac{\sin x}{\cos x}}{\sin x \cos x}$ $\qquad$ $\dfrac{1}{\sin x} - \dfrac{1}{\cos^2 x}$

$\dfrac{\cos^2 x - \sin x}{\sin x \cos^2 x}$ ✓

$\dfrac{\frac{\cos^2 x - \sin x}{\cos x}}{\sin x \cos x}$

$\dfrac{\cos^2 x - \sin x}{\cos x} \cdot \dfrac{1}{\sin x \cos x}$

$\dfrac{\cos^2 x - \sin x}{\sin x \cos^2 x}$ ✓