Multiple Choice:

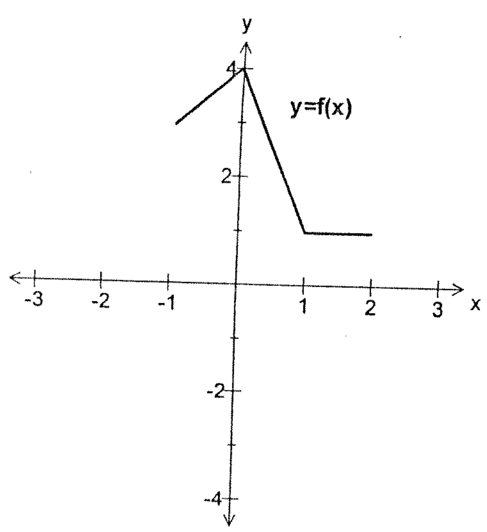
Midterm 2013

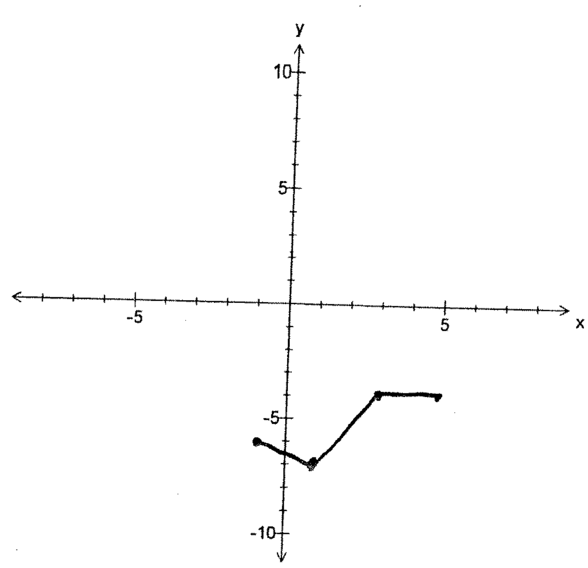
Answer Key

1. C
2. B
3. A
4. C
5. C
6. A
7. A
8. B
9. B
10. D
11. D
12. B
13. A
14. B
15. B
16. A
17. C
18. D
19. C
20. C
21. D
22. D
23. D
24. B
25. C
26. D

Answer **ALL** questions in the space provided. Show **ALL** necessary workings.

1. Given the graph of $f(x)$, sketch the graph of the transformed graph $y + 3 = -f\left(\frac{1}{2}(x-1)\right)$. (4 marks)





$y = -f\left(\frac{1}{2}(x-1)\right) - 3$

$(x, y) \to (2x+1, -y-3)$

x	y
-1	3
0	4
1	1
2	1

x'	y'
-1	-6
1	-7
3	-4
5	-4

2. Explain how the transformation described by $y = f\left(\frac{1}{4}x - 3\right)$ and $y = f\left(\frac{1}{4}(x-3)\right)$ are similar and how they are different. (2 marks)

$y = f\left(\frac{1}{4}\left(x - \frac{3}{4}\right)\right)$

same Horizontal stretch → 4

different Horizontal translation → $\frac{3}{4}$ vs. 3

3. Algebraically determine $f^{-1}(x)$ if $f(x) = \frac{1}{3}(x-5)^2 + 1$ and state any restrictions on the domain. (3 marks)

vertex$(5, 1)$

Restriction $x \geq 5$ (so that inverse is a function)

$y = \frac{1}{3}(x-5)^2 + 1$

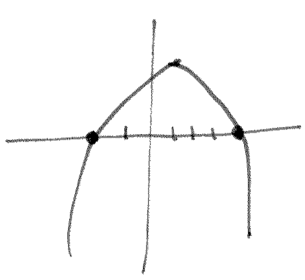
inverse → $x = \frac{1}{3}(y-5)^2 + 1$

$3(x-1) = (y-5)^2$

$\pm\sqrt{3(x-1)} + 5 = y$

$f^{-1}(x) = +\sqrt{3(x-1)} + 5, \quad x \geq 5$

4. Algebraically determine the domain and range of $y = \sqrt{-2x^2 + 4x + 16}$. (4 marks)



$-2x^2 + 4x + 16 = 0$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

vertex: $x = \frac{-b}{2a}$

$= 1$

$y = \sqrt{-2(1)^2 + 4(1) + 16}$

$= \sqrt{18}$

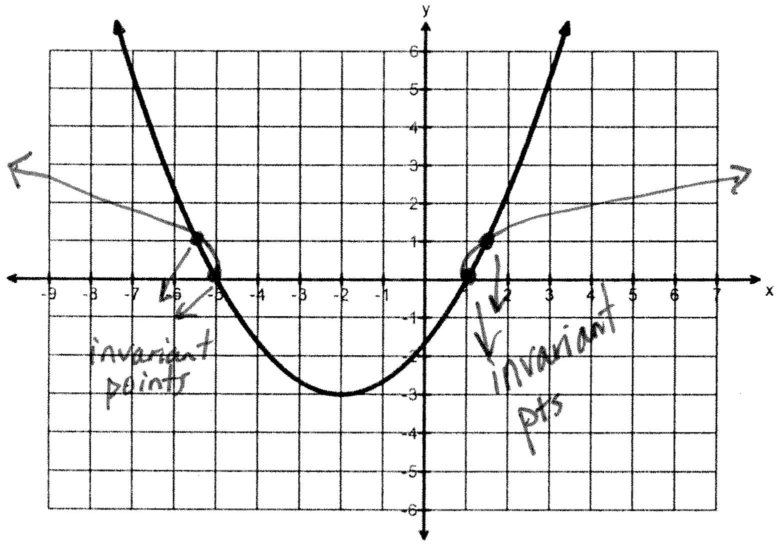
$= 3\sqrt{2}$

$(1, 3\sqrt{2})$

D: $[-2, 4]$

R: $[0, 3\sqrt{2}]$

5. The graph of $y = f(x)$ is shown.



(A) On the same grid, sketch the graph of the function $y = \sqrt{f(x)}$. Without stating the coordinates of the invariant points, clearly indicate the location of these points on the graph. (2 marks)

(B) State the domain and range of $y = \sqrt{f(x)}$. (2 marks)

D: $(-\infty, -5] \cup [1, \infty)$ R: $[0, \infty)$

(C) State where the function $y = \sqrt{f(x)}$ is undefined and justify your reasoning. (1 mark)

Undefined $-5 < x < 1$ since $f(x)$ is negative on this interval. & Can't take the square root of a negative number.

6. Algebraically determine all roots of $P(x) = 4x^3 - 4x^2 - 13x - 5$ (5 marks)

$4(-1)^3 - 4(-1)^2 - 13(-1) - 5 = 0$

$(x+1)$ is a factor

-1 is a root

-1	4	-4	-13	-5
(+)		-4	8	5
	4	-8	-5	0R

$4x^2 - 8x - 5 = 0$

$(2x+1)(2x-5) = 0$

$x = -1/2$ or $x = 5/2$

Roots $x = 1, -1/2, 5/2$

7. Determine the equation of the polynomial function shown below. (4 marks)

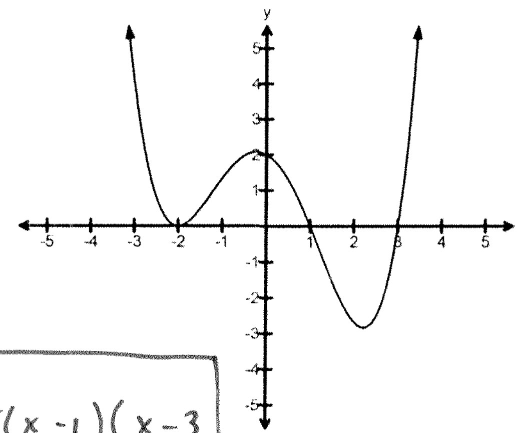
$f(x) = a(x+2)^2(x-1)(x-3)$

Sub. $(0,2)$ in for (x,y)

$2 = a(0+2)^2(0-1)(0-3)$

$2 = 12a$

$a = \frac{1}{6}$



$\boxed{f(x) = \frac{1}{6}(x+2)^2(x-1)(x-3)}$

8. Given $cot\theta = -\frac{5}{12}$, $90° \leq \theta \leq 180°$, determine the values of $sin\theta$ and $sec\theta$. (3 marks)

$\cot\theta = \frac{x}{y}$

$5^2 + 12^2 = r^2$

$r = 13$

$\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{12}{13}$

$\sec\theta = \frac{r}{x}$

$= \frac{-13}{5}$

9. Solve for Θ: 4 csc²Θ = –4 csc Θ + 15, where ~~$-2\pi \leq \Theta \leq 2\pi$~~. (4 marks)

$-\pi \leq \theta \leq \pi$

$4\csc^2\theta + 4\csc\theta - 15 = 0$

$(2\csc\theta - 3)(2\csc\theta + 5) = 0$

$\csc\theta = \frac{3}{2}$ or $\csc\theta = \frac{-5}{2}$

$\sin\theta = \frac{2}{3}$ $\sin\theta = \frac{2}{-5}$

$\theta = 0.730$ & 2.412 $\theta = -0.412, -2.73$