Chapter 3: Polynomial Functions

Section 3.1: Characteristics of Polynomial Functions

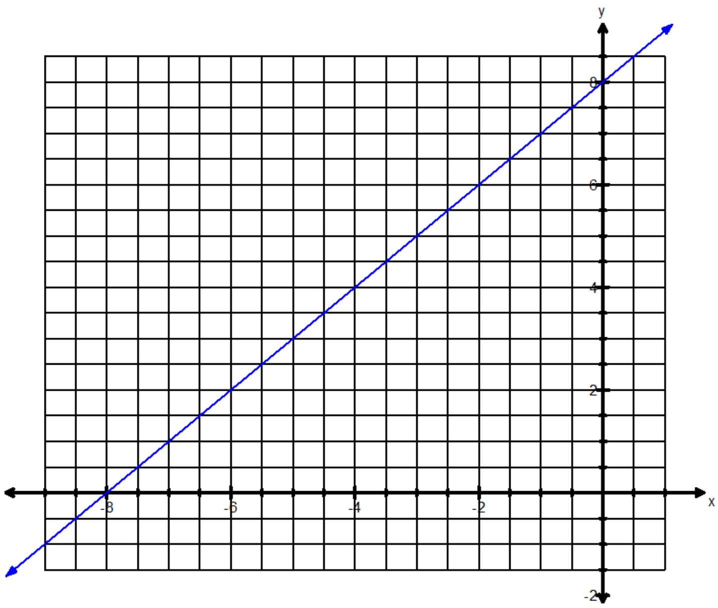
pg 107

Polynomial Function: a function of the form

$f(x) = a_nx^n + a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+....+a_2x^2+a_1x+a_0$

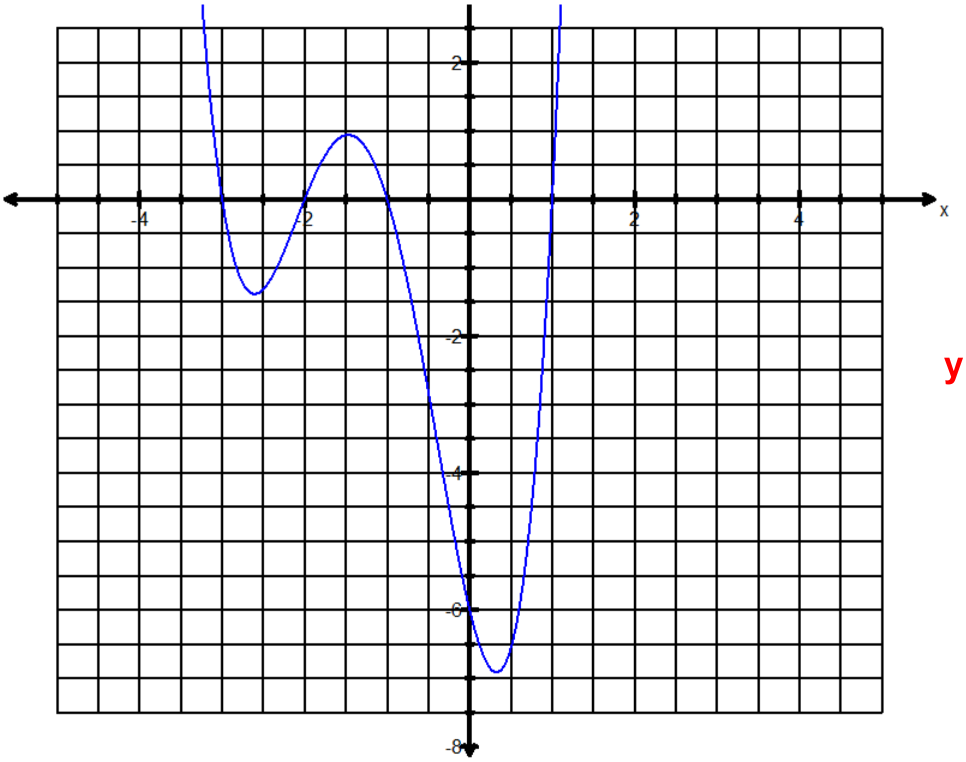
- where n is a whole number
- x is a variable
- the coefficients of a_n to a_o are real numbers

Key words: end behaviour, x-intercepts, degree, leading coefficient, # of turns, y-intercept



$y = x + 8$

General Shape	Degree	Lead. Coeff.	End Behav.	# Turns	# x-int.	y-int.



$y = x^4+5x^3+5x^2-5x-6$

General Shape	Degree	Lead. Coeff.	End Behav.	# Turns	# x-int.	y-int.

Chapter 3: Polynomial Functions

Section 3.1: Characteristics of Polynomial Functions

Example 1: (text pg 108)

Which functions are polynomials?

a) $g(x) = \sqrt{x} + 5$

No

b) $f(x) = 3x^4$

Yes

c) $y = |x|$

No

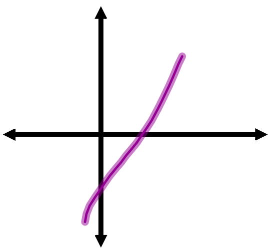
d) $y = 2x^3 + 3x^2 - 4x - 1$

Yes

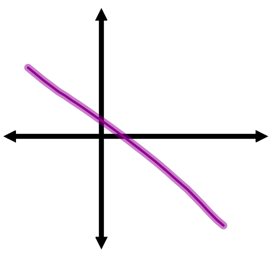
Characteristics of Polynomial Functions

Summary pg 109 in Text very important!!!

Degree 1: Linear Function
- 1 x-intercept



+ a



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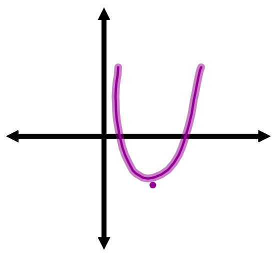
Degree 2: Quadratic Function
- 0,1 or 2 x-intercepts



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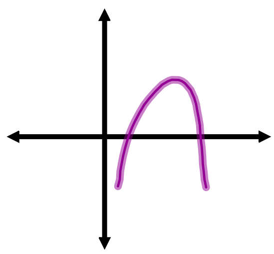

- a

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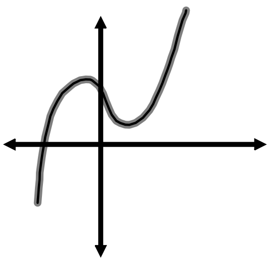
Degree 3: Cubic Function
- 1,2,or 3 x- intercepts



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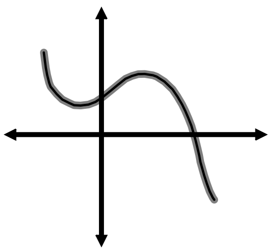

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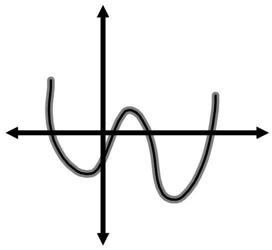
Degree 4: Quartic Function
- 0,1,2,3 or 4 x- intercepts



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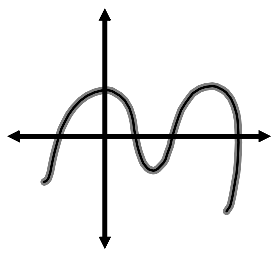
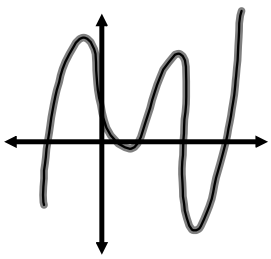


-a

Degree 5: Quintic Function
- 1,2,3, 4 or 5 x- intercepts



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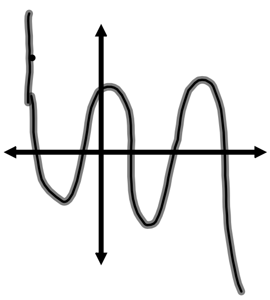
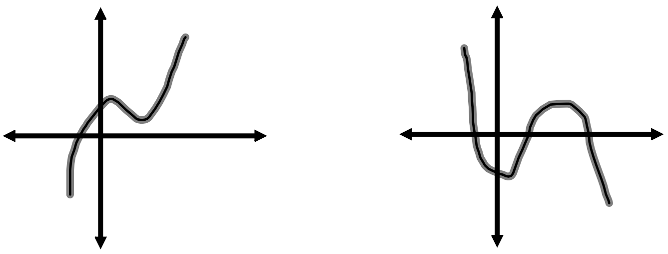


-a

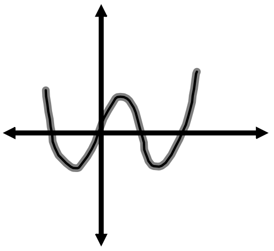
******Graphs of odd- degree polynomial functions:**

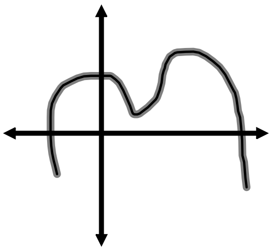
- when leading coefficient is positive, the graph extends down into quadrant 3 and up into quadrant 1
- when leading coefficient is negative, the graph extends up into quadrant 2 and down into quadrant 4
- Domain $\{x/x \in R\}$
- Range $\{y/y \in R\}$
- no max or min points



*****Graphs of even-degree polynomial functions:**

- when the leading coefficient is positive, the graph extends up into quadrant 2 and up into quadrant 1
- when the leading coefficient is negative, the graph extends down into quadrant 3 and down into quadrant 4
- domain {x/x$\in$ R}
- range depends on the max or min value of the function



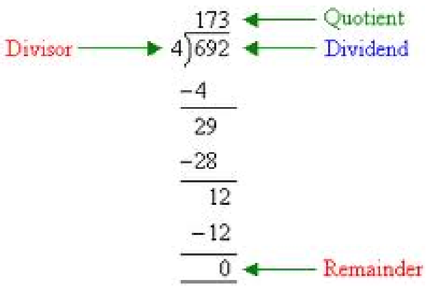


Questions 1, 2, 3, 4 pg 114-115

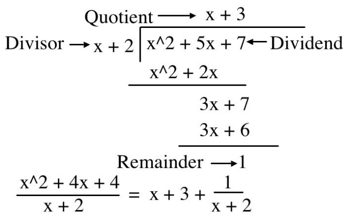
Questions 1, 2, 3, 4 pg 114-115

Section 3.2: The Remainder Theorem

Long Division



Long Division Polynomials



Section 3.2: The Remainder Theorem

Example 1 a) Divide

$$\begin{array}{r} x^2+6x+8 \\ x+1\overline{)\,x^3+7x^2+14x+8} \\ (-)\;\underline{x^3+x^2} \\ 6x^2+14x \\ (-)\;\underline{6x^2+6x} \\ 8x+8 \\ (-)\;\underline{8x+8} \\ 0R \end{array}$$

ANS: x^2+6x+8

Section 3.2: The Remainder Theorem

Ex 1b: pg 120 in text

a) Divide the polynomial $P(x) = 5x^3 + 10x - 13x^2 - 9$, by *x* - 2. Express the result in the form

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$ where Q(x) is the quotient, x - a is the binomial divisor and R is the remainder

$$\begin{array}{r} 5x^2 - 3x + 4 \\ x-2\overline{)5x^3 - 13x^2 + 10x - 9} \\ (-)\ 5x^3 - 10x^2 \qquad\qquad \\ \hline -3x^2 + 10x \qquad \\ (-)\ -3x^2 + 6x \qquad \\ \hline 4x - 9 \\ -\ 4x - 8 \\ \hline -1R \end{array}$$

ANS: $5x^2 - 3x + 4 - \frac{1}{x-2}$

i) Identify the restrictions on the variable.

$x - 2 \neq 0$

$x \neq 2$

ii) Write the corresponding statement that can be used to check the divsion.

$(x-2)(5x^2-3x+4) - 1$

iii) Verify your answer.

Example 1c:

Divide: $P(x)=x^4-2x^3+x^2-3x+4$ by $x-1$

$$
\begin{array}{r}
x^3-x^2-3 \\
x-1\overline{)\,x^4-2x^3+x^2-3x+4} \\
(-)\;\underline{x^4-x^3} \\
-x^3+x^2 \\
(-)\;\underline{-x^3+x^2} \\
0-3x+4 \\
(-)\;\underline{-3x+3} \\
1R
\end{array}
$$

b) Identify the restrictions on the variable.

$x \neq 1$

c) Write the corresponding statement that can be used to check the divsion.

Note: Be careful if a polynomial is missing a term!

#1, 2, 3 pg 124

Synthetic Division - alternate method that allows you to divide without writing variables and requires fewer calculations

Example 1: Pg 122 in text

$2x^3 + 3x^2 - 4x + 15$ by $x + 3$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -4 & 15 \\ (+) & & -6 & 9 & -15 \\ \hline & 2 & -3 & 5 & 0R \end{array}$$

$\downarrow$ x^2 $\downarrow$ x $\downarrow$ constant

ANS: $2x^2 - 3x + 5$

Example 1b): Divide using synthetic division

$P(x) = 5x^3 + \underline{10x - 13x^2} - 9$ by $x - 2$

$$\begin{array}{r|rrrr} 2 & 5 & -13 & 10 & -9 \\ (+) & & 10 & -6 & 8 \\ \hline & 5 & -3 & 4 & -1R \end{array}$$

ANS: $5x^2 - 3x + 4 - \dfrac{1}{x-2}$

Pg 124

#1, 3ad, 4a, c, e

*

Remainder Theorem - when a polynomial, P(x), is divided by x - a, the remainder is P(a)

Example 1: *pg 123 in text*

Use the remainder theorem to determine the remainder when $P(x) = x^3 - 10x + 6$ is divided by x + 4.

$P(-4) = (-4)^3 - 10(-4) + 6$

$= -18$

Section 3.3: The Factor Theorem

Factor Theorem: a polynomial, P(x), has a factor x - a if and only if P(a)= 0

Example 1: *pg 128*

Which binomials are factors of the polynomial $P(x) = x^3 - 3x^2 - x + 3$?

a) x - 1 $P(1) = (1)^3 - 3(1)^2 - 1 + 3 = 0$ $\therefore$ $x-1$ is a factor

b) x + 1 $P(-1) = (-1)^3 - 3(-1)^2 + 1 + 3 = 0$

$\therefore$ $x+1$ is a factor

c) x + 2

$P(-2) = -15$

integral zero theorem:

- if *x*- *a* is a factor of a polynomial function P(x), with integral coefficients, then *a* is a factor of the constant term of P(x)

Example 2: *pg 129*

a) factor $2x^3 - 5x^2 - 4x + 3$

- find a factor by evaluating P(*x*) for values of *x* that are factors of 3
- test ± 1 and ± 3 using the remainder theorem

$P(1) = 2(1)^3 - 5(1)^2 - 4(1) + 3 \neq 0$

$P(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3 = 0$

$\therefore$ x+1 is a factor

- use synthetic division to find the other factors

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & 3 \\ & & -2 & 7 & -3 \\ \hline & 2x^2 & -7x & +3 & \text{OR} \end{array}$$

$(x+1)(2x^2 - 7x + 3)$

$(x+1)(2x-1)(x-3)$ Final Ans

Example 2b: Factor $x^3 - 4x^2 - 11x + 30$

$P(2) = 0 \therefore x - 2$ is a factor

$$
\begin{array}{r|rrrr}
2 & 1 & -4 & -11 & 30 \\
(+) & & 2 & -4 & -30 \\
\hline
 & 1 & -2 & -15 & 0
\end{array}
$$

$x^2 - 2x - 15$

$(x-5)(x+3)$

Final Ans:

$(x-2)(x-5)(x+3)$

Example 3: *pg 130*

Factor Fully: $x^4 - 5x^3 + 2x^2 + 20x - 24$

$P(2)=0 \therefore x-2$ is a factor

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 2 & 20 & -24 \\ (+) & & 2 & -6 & -8 & 24 \\ \hline & 1 & -3 & -4 & 12 & \end{array}$$

OR

$$\boxed{x^3 - 3x^2 - 4x + 12}$$

* New factoring technique

$x^3 - 3x^2 \mid -4x + 12$

$x^2(x-3) - 4(x-3)$

$(x^2-4)(x-3)$

$(x-2)(x+2)(x-3)$

Final Ans:

$(x-2)(x+2)(x-3)(x-2)$

Example 3b: *pg 130*

Factor Fully: $x^4 - 3x^3 - 7x^2 + 15x + 18$

$P(-1) = 0$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & -7 & 15 & 18 \\ & & -1 & 4 & 3 & -18 \\ \hline & 1 & -4 & -3 & 18 & 0R \end{array}$$

$$\boxed{x^3 - 4x^2 - 3x + 18}$$

$P(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -3 & 18 \\ (+) & & -2 & 12 & -18 \\ \hline & 1 & -6 & 9 & 0R \end{array}$$

$$\boxed{x^2 - 6x + 9}$$

$(x-3)(x-3)$

Final Ans:

$(x-3)^2(x+1)(x+2)$

pg 134 - #5 d,e (grouping!!!), 6e, 7
pg 124 - #8, 9, 10, 11, 12, 14***, 15

pg 153 1, 2, 4 a) b), 5, 6, 7ab, 8, 10

Example 4: When $3x^3+mx^2+nx+2$ is divided by x + 2, the remainder is 8. When the same polynomial is divided by x - 1, the remainder is 2. Determine the values of m and n.

$$3(-2)^3+m(-2)^2+n(-2)+2=8$$

$$\boxed{4m-2n=30}$$

$$3(1)^3+m(1)^2+n(1)+2=2$$

$$x-4 \Rightarrow \boxed{m+n=-3}$$

$$4m-2n=30$$
$$(+)\ -4m-4n=12$$
$$-6n=42$$
$$\boxed{n=-7}$$

$$m+n=-3$$
$$m+(-7)=-3$$
$$\boxed{m=4}$$

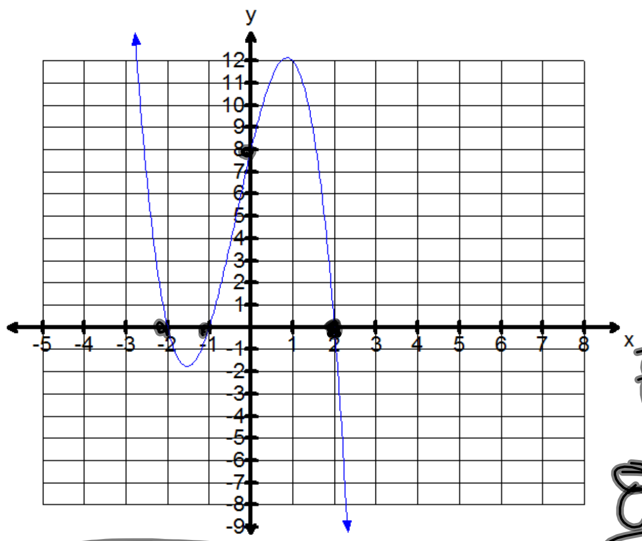
Section 3.4: Equations and Graphs of Polynomial Functions

Example 1: Solve x (x+5) (x-7) = 0

$\boxed{x=0}$ or $x+5=0$ or $x-7=0$

$\boxed{x=-5}$ $\boxed{x=7}$

Example 2: Write an equation for the graph below



$y=a(x-r)(x-s)(x-t)$

$y=a(x-2)(x+1)(x+2)$

Subst $(0,8)$

$8=a(0-2)(0+1)(0+2)$

$8=-4a$

$\boxed{a=-2}$

$\boxed{y=-2(x-2)(x+1)(x+2)}$

Multiplicity

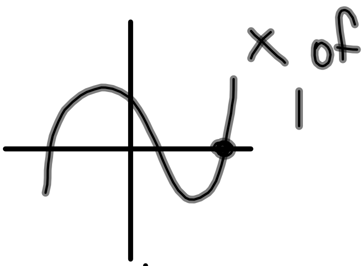
The zeros of any polynomial functions y = f(x) correspond to the x-intercepts of the graph and to the roots of the corresponding equation, f(x) = 0

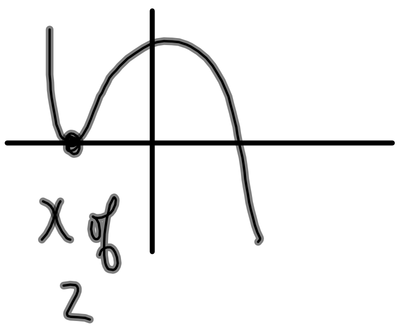
multiplicity (of a zero)

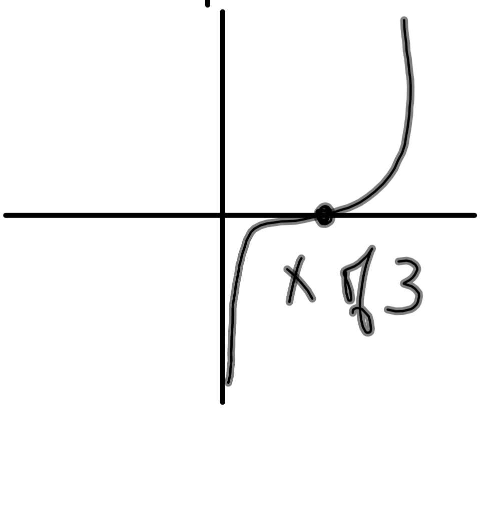
- the number of times a zero of a polynomial function occurs
- If a polynomial has a factor x-a, then x = a is a zero of multiplicity,n

Ex: The function $f(x) = (x-1)^2(x+2)$ has a zero of multiplicity 2 at x = 1

- the shape of the graph of a function close to a zero depends on its multiplicity



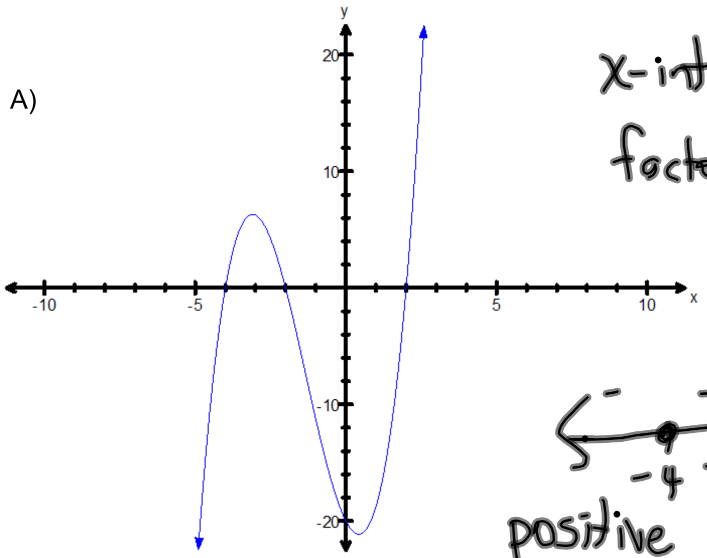




#1-4 pg 147

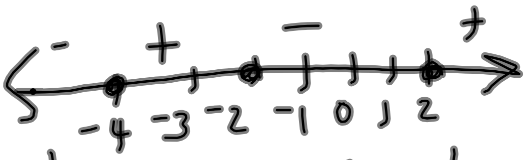
Example 1 (pg 138): For each graph of a polynomial function, determine
- the least possible degree 3
- the sign of the leading coefficient +
- the x-intercepts and the factors of the function with least possible degree
- the intervals where the function is positive and the intervals where it is negative
- Write a possible equation

A)



x-ints ⇒ $2, -2, -4$

factors ⇒ $(x-2), (x+2), (x+4)$



positive

$x > 2, -4 < x < -2$

Neg

$(-2,2) \cup (-\infty, -$

$y = a(x-2)(x+2)(x+4)$

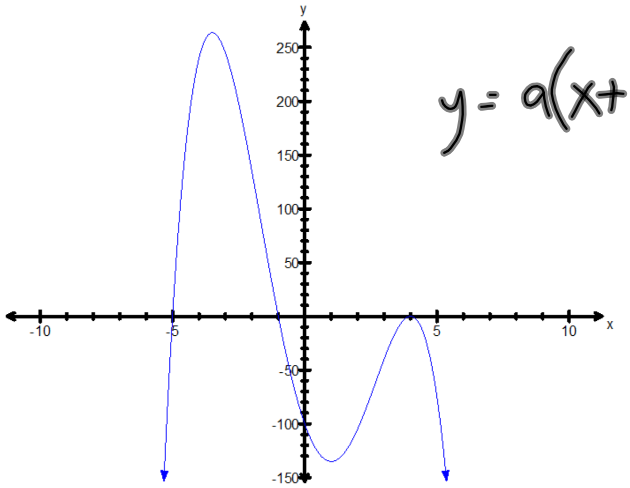
$-20 = a(0-2)(0+2)(0+4)$

$-20 = -16a$

$5/4 = a$

$y = 5/4(x-2)(x+2)(x+4)$

B)



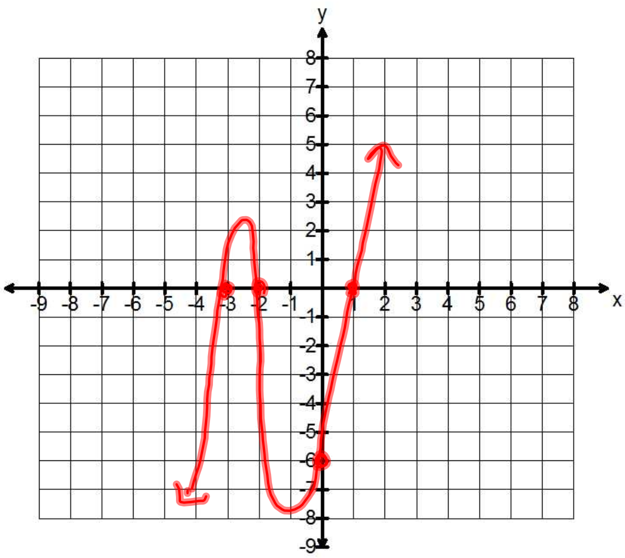
$y = a(x+5)(x+1)(x-4)^2$

$x < -5$

Example 2*(pg 140 in text)*: Sketch the graph of each polynomial function.

a) $y = (x-1)(x+2)(x+3)$

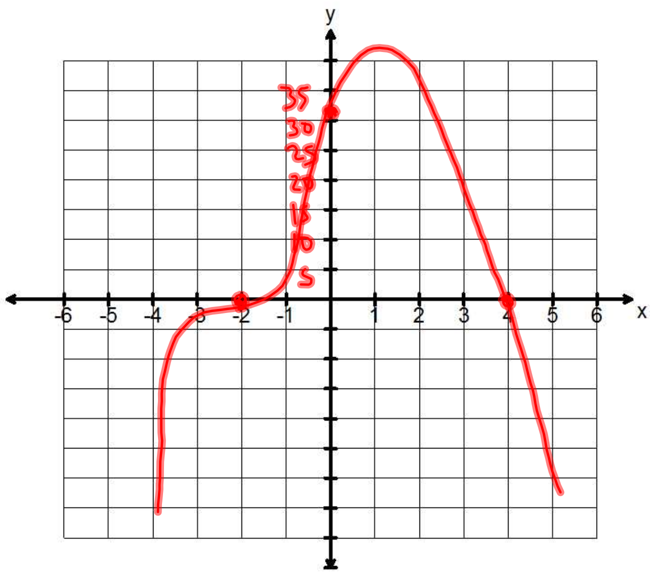
Degree	3
Leading Coefficient	+
End Behaviour	up Q1, down Q3
Zeros/x-intercepts	1, -2, -3
y-intercept (let x=0)	-6


y
x
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-9 -8 -7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8

b) Graph $f(x) = -(x+2)^3(x-4)$

$-(8)(-4)$

Degree	4
Leading Coefficient	−
End Behaviour	down Q3 down Q4
Zeros/x-intercepts	-2 (x 3), 4
y-intercept	32



c) $y = -2x^3+6x-4$

$P(1)=0$

$$\begin{array}{r|rrrr} 1 & -2 & 0 & 6 & -4 \\ & & -2 & -2 & 4 \\ \hline & -2 & -2 & 4 & 0R \end{array}$$

$-2x^2-2x+4$

$-2(x^2+x-2)$

$-2(x+2)(x-1)$

$y=-2(x+2)(x-1)(x-1)$

$=-2(x+2)(x-1)^2$

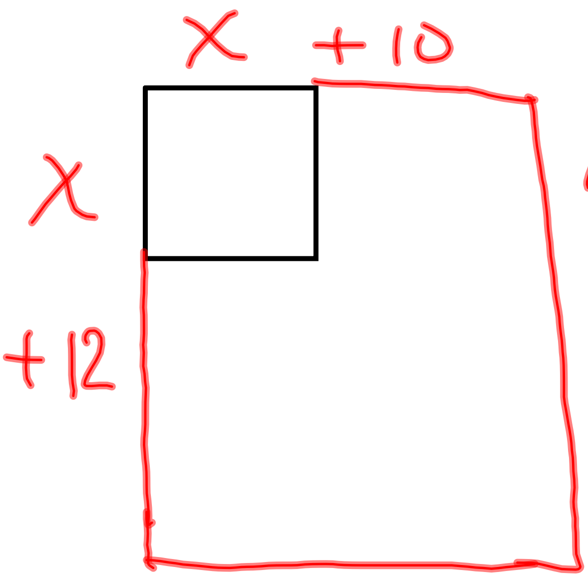
#1-4 pg 147

pg 148 - 149
#5, 7, 8, 9, 10

pg 153-154 - all questions except #12

Recall from Math 2200

The length of two opposite sides of a square is increased by 10m. The length of the other two sides is increased by 12m. The area of the resulting rectangle is $675m^2$. What is the length of the sides of the original square?



$A = l \times w$

$(x+10)(x+12) = 675$

$x^2 + 22x + 120 - 675 = 0$

$x^2 + 22x - 555 = 0$

$(x+37)(x-15) = 0$

$x+37=0$ or $x-15=0$

$\cancel{x=-37}$ or $x=15$

reject

Section 3.4 Cont'd.....

Application Questions

Example 1: Bill is preparing to make an ice sculpture. He has a block of ice that is 3ft by 4ft by 5ft. He wants to reduce the size of the block of ice by removing the same amount from each of the three dimensions. Bill wants the volume of the block to be 24ft^3.

A) Write a polynomial function to model this
B) How much should he remove from each dimension?

A) x = the reduction in each dimension

$$(3-x)(4-x)(5-x)=24$$

$$-x^3+12x^2-47x+60=24$$

$$-x^3+12x^2-47x+36=0$$

$$-1(x^3-12x^2+47x-36)=0$$

$$P(1)=0$$

$$1\,|\;\; 1 \quad -12 \quad 47 \quad -36$$

$$\boxed{x^2-11x+36}$$ Not factorable

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{11\pm\sqrt{(-11)^2-4(1)(36)}}{2(1)}$$

No real soln:

B) 1 ft needs to be removed.

Example 2: Three consecutive integers have a product of -210.
A) Write a polynomial function to model this.
B) What are the integers?

$x \rightarrow 1^{st}$ #
$x+1 \rightarrow 2^{nd}$ #
$x+2 \rightarrow 3^{rd}$ #

A) $(x)(x+1)(x+2) = -210$

$(x^2+x)(x+2) = -210$

$x^3+2x^2+x^2+2x = -210$

$\boxed{x^3+3x^2+2x+210=0}$

$P(-7)=0$

$-7 \,|\, 1 \quad 3 \quad 2 \quad 210$

$\boxed{x^2-4x+30}$

$x = \dfrac{4 \pm \sqrt{(-4)^2-4(1)(30)}}{2(1)}$

No Real Soln

B) $-7, -6, -5$

pg 150-151 12, 13, 14, 15,16
pg 154 #15