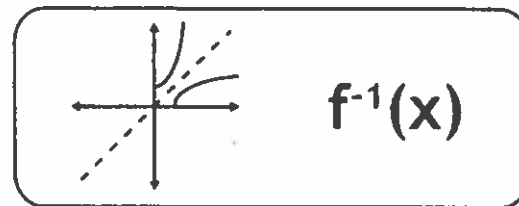


# Transformations and Operations

## LESSON THREE - Inverses

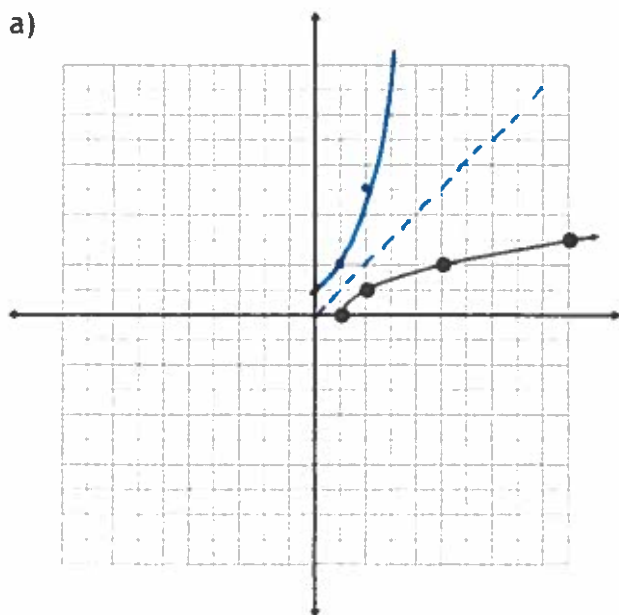
### Lesson Notes



### Example 2

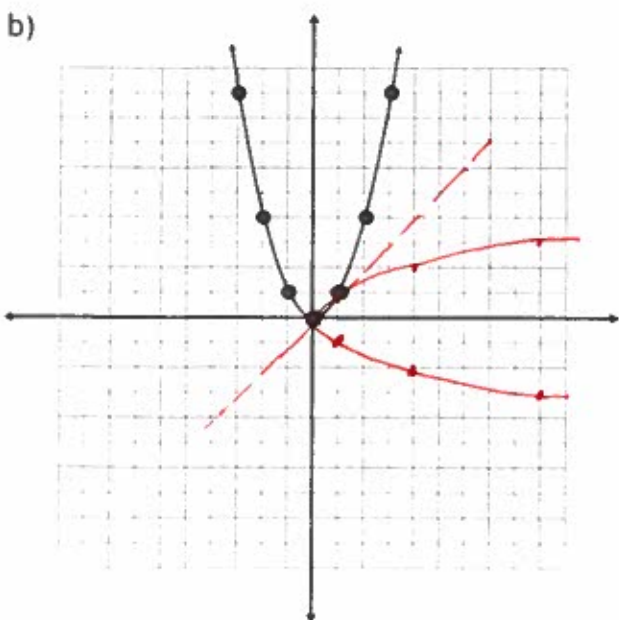
For each graph, answer parts (i - iv).

Domain and Range



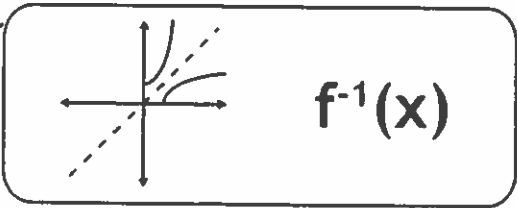
- i) Draw the graph of the inverse.
- ii) State the domain and range of the original graph.  
 $\{x \mid x \geq 1, x \in \mathbb{R}\}$   
 $\{y \mid y \geq 0, y \in \mathbb{R}\}$
- iii) State the domain and range of the inverse graph.  
 $\{x \mid x \geq 0, x \in \mathbb{R}\}$   
 $\{y \mid y \geq 1, y \in \mathbb{R}\}$
- iv) Can the inverse be represented with  $f^{-1}(x)$ ?

yes



- i) Draw the graph of the inverse.
- ii) State the domain and range of the original graph.  
 $\{x \mid x \in \mathbb{R}\}$   
 $\{y \mid y \geq 0, y \in \mathbb{R}\}$
- iii) State the domain and range of the inverse graph.  
 $\{x \mid x \geq 0, x \in \mathbb{R}\}$   
 $\{y \mid y \in \mathbb{R}\}$
- iv) Can the inverse be represented with  $f^{-1}(x)$ ?

NO

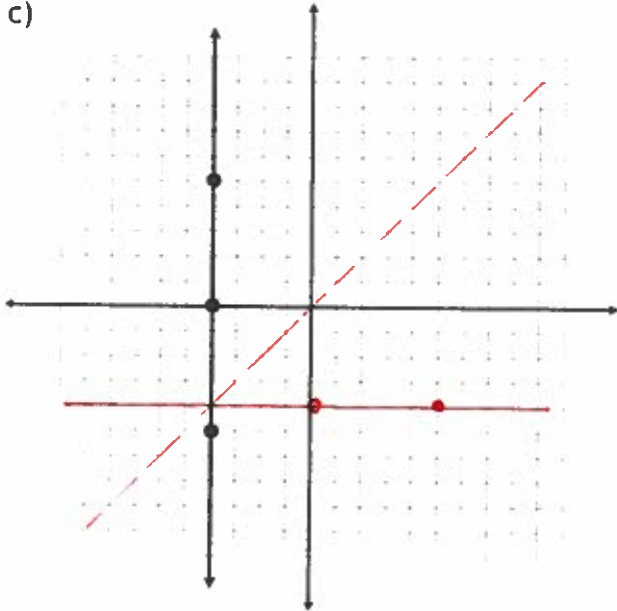


# Transformations and Operations

## LESSON THREE - Inverses

### Lesson Notes

c)

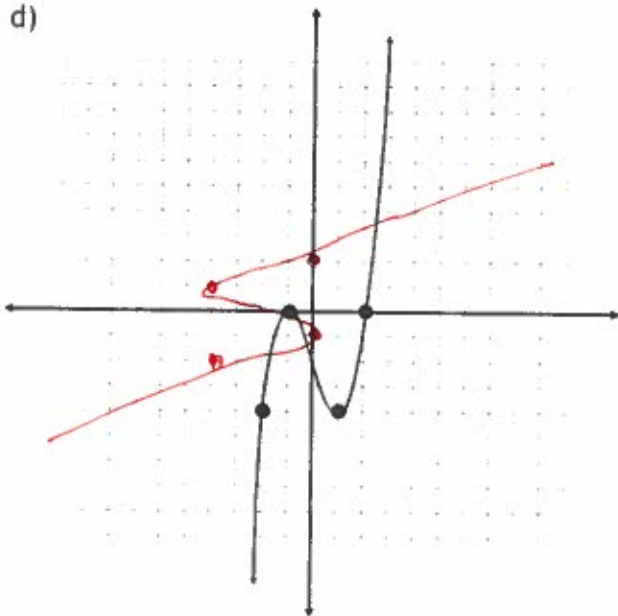


Domain and Range

- i) Draw the graph of the inverse.
- ii) State the domain and range of the original graph.  
 $x = -4$   
 $y \in \mathbb{R}$
- iii) State the domain and range of the inverse graph.  
 $x \in \mathbb{R}$   
 $y = -4$
- iv) Can the inverse be represented with  $f^{-1}(x)$ ?

yes

d)



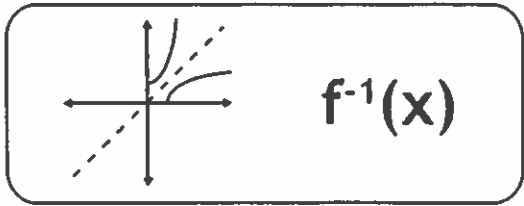
- i) Draw the graph of the inverse.
- ii) State the domain and range of the original graph.  
 $x \in \mathbb{R}$   
 $y \in \mathbb{R}$
- iii) State the domain and range of the inverse graph.  
 $x \in \mathbb{R}$   
 $y \in \mathbb{R}$
- iv) Can the inverse be represented with  $f^{-1}(x)$ ?

NO

# Transformations and Operations

## LESSON THREE - Inverses

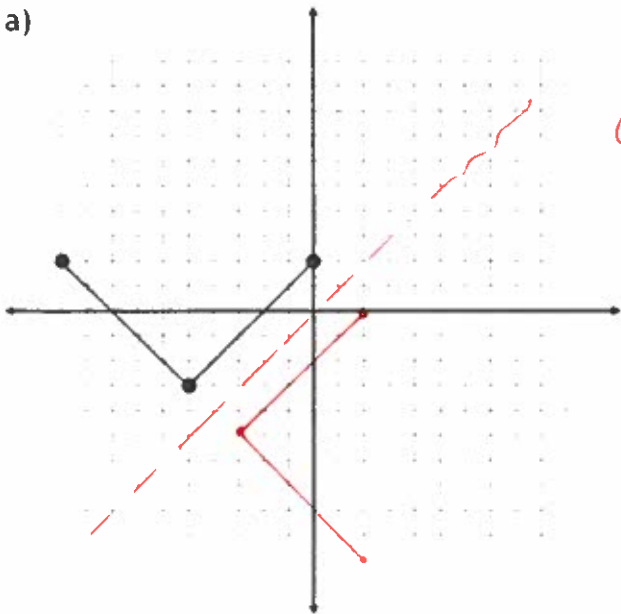
### Lesson Notes



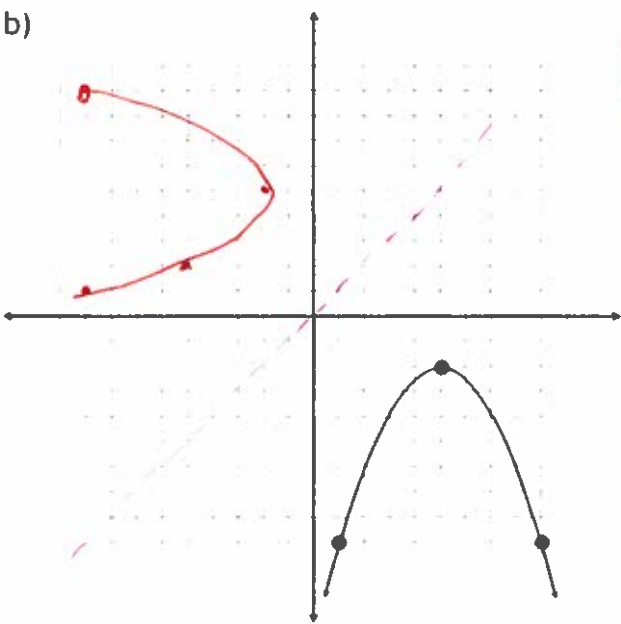
### Example 3

For each graph, draw the inverse. How should the domain of the original graph be restricted so the inverse is a function?

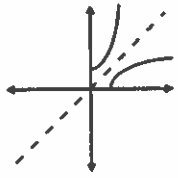
Domain restrictions



*Restricted Domain of original*  
 $x \geq -5$       or       $-10 \leq x \leq -5$



*Restricted Domain of original*  
 $x \geq 5$       or       $x \leq 5$



$f^{-1}(x)$

# Transformations and Operations

## LESSON THREE - Inverses

### Lesson Notes

#### Example 4

Find the inverse of each linear function algebraically. Draw the graph of the original function and the inverse. State the domain and range of both  $f(x)$  and its inverse.

Inverses of linear functions

a)  $f(x) = x - 3$

$$y = x - 3$$

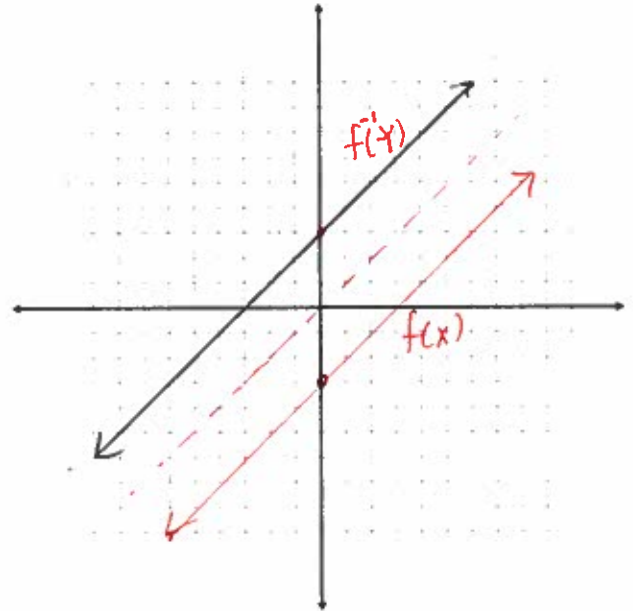
$$x = y - 3$$

$$x + 3 = y$$

$$f^{-1}(x) = x + 3$$

Domain  $\{x | x \in \mathbb{R}\}$     Range  $\{y | y \in \mathbb{R}\}$

For Both



b)  $f(x) = -\frac{1}{2}x - 4$

$$y = -\frac{1}{2}x - 4$$

$$x = -\frac{1}{2}y - 4$$

$$x + 4 = -\frac{1}{2}y$$

$$-2(x + 4) = y$$

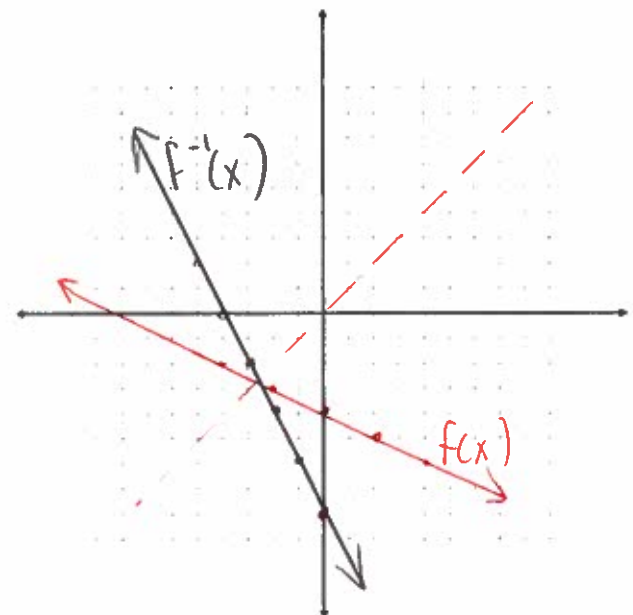
$$-2x - 8 = y$$

$$f^{-1}(x) = -2x - 8$$

Domain  $x \in \mathbb{R}$

Range  $x \in \mathbb{R}$

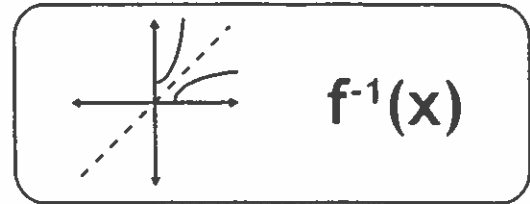
for both!



# Transformations and Operations

## LESSON THREE - Inverses

### Lesson Notes



#### Example 5

Find the inverse of each quadratic function algebraically. Draw the graph of the original function and the inverse. Restrict the domain of  $f(x)$  so the inverse is a function.

inverses of quadratic functions

a)  $f(x) = x^2 - 4$

$$y = x^2 - 4$$

$$x = y^2 - 4$$

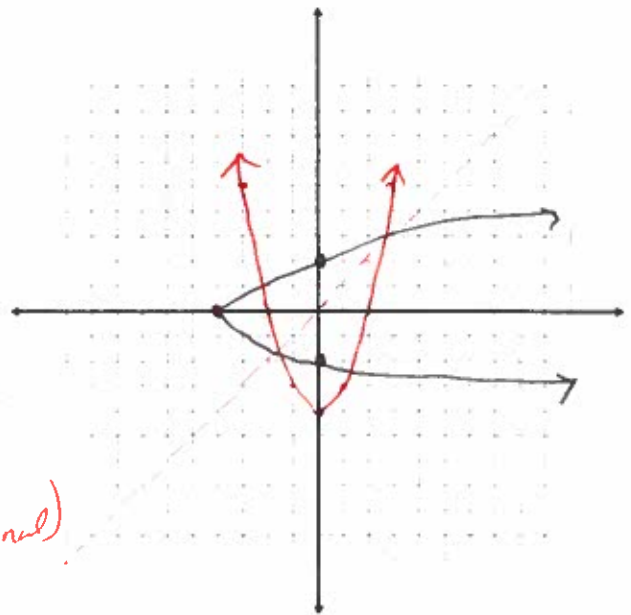
$$x + 4 = y^2$$

$$y = \pm \sqrt{x + 4}$$

top half  
bottom half.

Restriction  
vertex:  $(0, -4)$   
therefore  
restriction  
 $x \geq 0$

therefore  
 $f^{-1}(x) = \sqrt{x + 4}$   
Domain? (change of original)



b)  $f(x) = -(x + 3)^2 + 1$

$$y = -(x + 3)^2 + 1$$

inverse

$$x = -(y + 3)^2 + 1$$

$$x - 1 = -(y + 3)^2$$

$$-(x - 1) = (y + 3)^2$$

$$\pm \sqrt{-(x - 1)} = y + 3$$

$$\pm \sqrt{-(x - 1)} - 3 = y$$

Restriction  
vertex  $(-3, 1)$   
 $x \geq -3$

$$f^{-1}(x) = \sqrt{-(x - 1)} - 3$$

