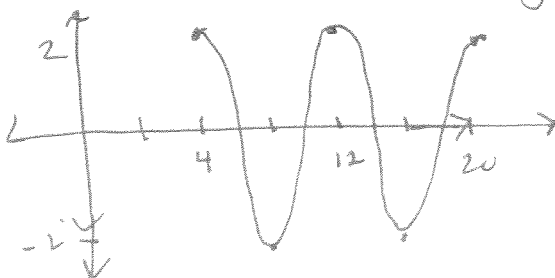


Part A: Circle the letter that corresponds to the correct answer.

1. What is the equation of the sinusoidal axis of $y = -2 \sin 3(x - \pi) + 5$?
- a. $y = -2$ b. $y = 3$ c. $y = \pi$ **d. $y = 5$**
2. What is the phase shift of: $y = \sin(3x - 120^\circ)$ $3(x - 40^\circ)$
- a. -40° **b. 40°** c. -120° d. 120°
3. What is the period of: $y = \cos 5(x - \pi) - 3$? $\frac{1}{5} \cdot 2\pi = \frac{2\pi}{5}$
- a. $\frac{\pi}{5}$ **b. $\frac{2\pi}{5}$** c. 5π d. 10π
4. Which of the following functions has vertical displacement of -5 ?
- a. $(y + 5) = \sin 2(x - 30)$** b. $y = \frac{1}{5} \cos(x - 100)$
- c. $-\frac{1}{5}(y + 1) = \sin(x + 90)$ d. $(y - 1) = \cos(x - 5)$
5. What is the range of the function: $y = 3 \sin(x - 30) - 2$?
- a. $\{y \mid -5 \leq y \leq 1, y \in \mathbb{R}\}$** b. $\{y \mid -5 \geq y \geq 1, y \in \mathbb{R}\}$ $-2 + 3 = 1$ $-2 - 3 = -5$
- c. $\{y \mid 27 \leq y \leq 33, y \in \mathbb{R}\}$ d. $\{y \mid 27 \geq y \geq 33, y \in \mathbb{R}\}$
6. The following equation represents the height of a porpoise as it swims along the side of a boat. x represents time in seconds and y represents meters above the water. What is the porpoise's height at 16 seconds? $y = 2 \cos 45(x - 4)$
- a. -2** b. -5 c. 0 d. 2



$$y = 2 \cos 45(16 - 4)$$

$$= 2 \cos 45(12)$$

$$\frac{1}{45} \cdot 360$$

8 seconds

7. What is the function that produces the following mapping rule $(x, y) \rightarrow (2x - 90, -\frac{1}{2}y)$

$$\frac{1}{2}(x+90)$$

a. $y = -2 \cos 2(x - 90)$

b. $y = -2 \cos \frac{1}{2}(x - 90)$

c. $y = -\frac{1}{2} \cos 2(x + 90)$

d. $y = -\frac{1}{2} \cos \frac{1}{2}(x + 90)$

8. A sinusoidal function has a maximum value at $(\frac{\pi}{6}, 6)$. It has an adjacent minimum value located to the right located at $(\frac{5\pi}{3}, 2)$. What is the period of the function?

$$\left(\frac{5\pi}{3} - \frac{\pi}{6}\right) \times 2 = 3\pi$$

a. $\frac{3\pi}{2}$

b. π

c. 2π

d. 3π

9. Where are the asymptotes of $y = \tan x$ located?

a. $x = \pi k, k \in I$

b. $x = \frac{\pi}{2} k, k \in I$

c. $x = \pi + \frac{\pi}{2} k, k \in I$

d. $x = \frac{\pi}{2} + \pi k, k \in I$

10. Which of these equations is NOT the equation of an asymptote of $y = \tan x$?

a. $x = \frac{-3\pi}{2}$

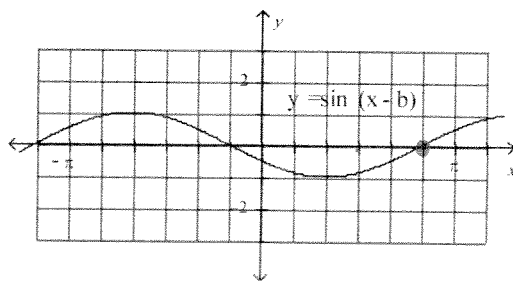
b. $x = \frac{-5\pi}{2}$

c. $x = \frac{5\pi}{2}$

d. $x = \frac{7\pi}{3}$

$$\frac{\pi}{2} + \pi k, k \in I = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

11. This graph is the image of $y = \sin x$ after a phase shift. Which value below could represent the phase shift?

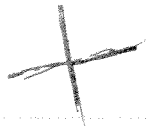


a. $\frac{5\pi}{6}$

b. $\frac{-5\pi}{6}$

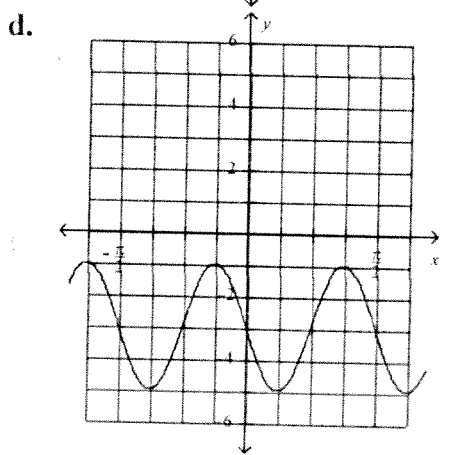
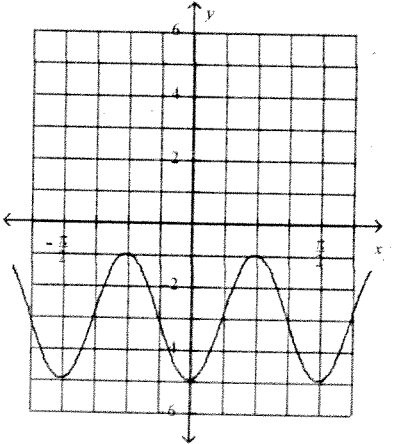
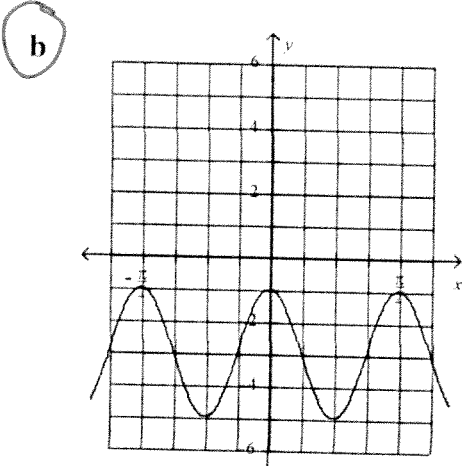
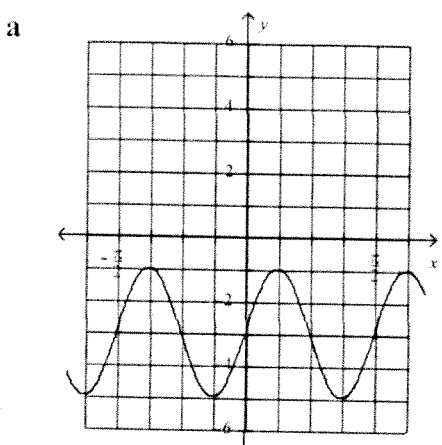
c. $\frac{-17\pi}{6}$

d. $\frac{7\pi}{6}$



Period
 $\frac{1}{4}(2\pi) = \frac{\pi}{2}$

12. Which graph below can be described by $y = 2 \sin 4\left(x + \frac{\pi}{8}\right) - 3$?



13. Suppose the function $y = 11 \cos \frac{2\pi}{7}(x - 3.5) + 15$ models the height, y metres, of a seat on a Ferris wheel at any time x minutes after the wheel begins to rotate. How long does it take the Ferris wheel to complete one full rotation?

- a 11 min
- b 7 min**
- c 15 min
- d $\frac{1}{7}$ min

14. The minimum value of the function $f(\theta) = a \cos b(\theta - c) + d$, where $a > 0$, can be expressed as

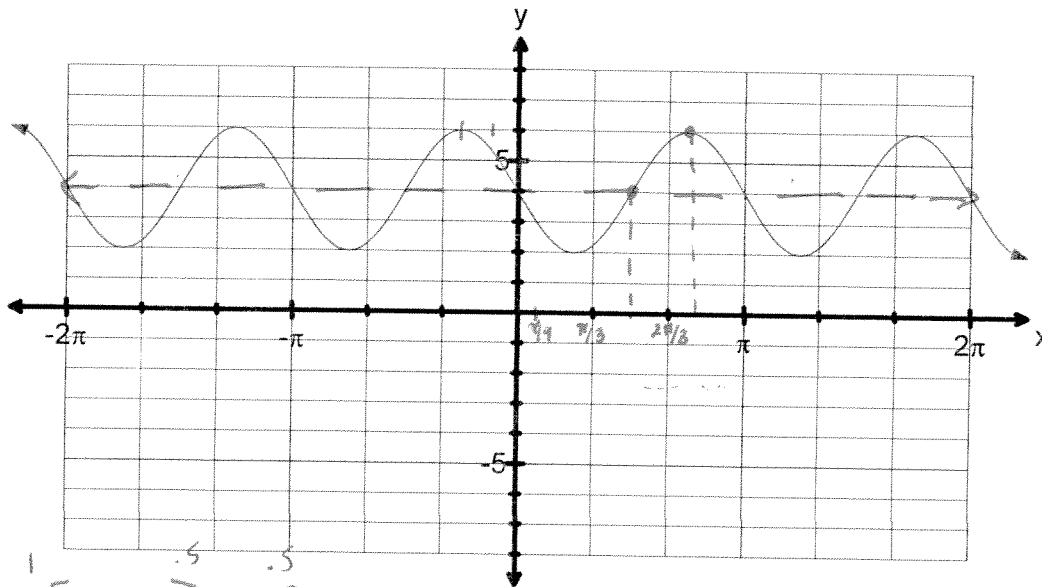
- a. $a - d$
- b. $a - d - c$
- c. $d - a$**
- d. $\frac{d - a}{b}$

15. Colin is investigating the effect of changing the values of the parameters a , b , c , and d in the equation $y = a \sin b(\theta - c) + d$. He graphed the function $f(x) = \sin \theta$. He then determined that the transformation that does not change the x -intercepts is described by

- a. $g(\theta) = 2 \sin \theta$ b. $h(\theta) = \sin 2\theta$
 c. $r(\theta) = \sin(\theta + 2)$ d. $s(\theta) = \sin \theta + 2$

Part B. Answer all questions in the space provided. Show all necessary workings.

16. Write the equation for the sine and ~~cosine~~ function for the graph shown below. (4)



Handwritten equations for the graph:

$$y = 2 \sin 2 \left[x - \frac{\pi}{2} \right] + 4$$

$$y = 2 \cos 2 \left[x - \frac{7\pi}{9} \right] + 4$$

Handwritten equation for the graph:

$$y = 2 \cos 2 \left[x + \frac{2\pi}{9} \right] + 4$$

17. For the function $y = 2 \sin \left(3x + \frac{\pi}{2} \right) + 6$ determine the following: (3 marks)

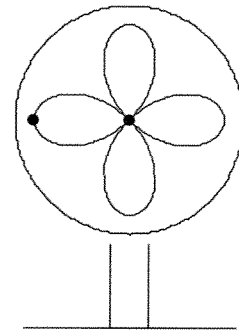
- a) Period 2π/3
 b) Amplitude 2
 c) Phase Shift -π/6
 d) Range 4 ≤ y ≤ 8
 e) Maximum Value 8
 f) Minimum Value 4

Handwritten equation for question 17:

$$y = 2 \sin 3 \left(x + \frac{\pi}{6} \right) + 6$$

17. A mark on the tip of one blade of a table fan is 24 cm from the axle, and the axle is 30 cm above the table. The fan is turned on when the mark has the position shown. The fan completes one clockwise rotation in $\frac{1}{4}$ s. Write a sinusoidal function that represents the height of the mark, y centimetres, above the table x seconds after the fan is turned on.

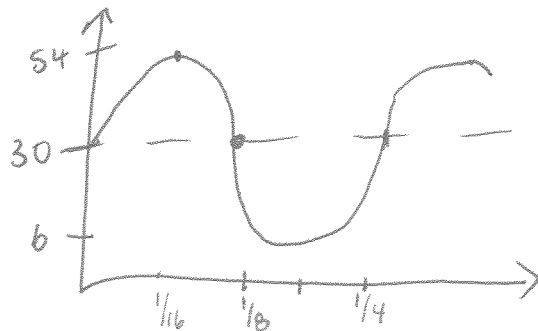
[4 marks]



$$y = 24 \cos 8\pi(x - \frac{1}{16}) + 30$$

$$y = 24 \sin 8\pi x + 30$$

$$HS = \frac{\frac{1}{4}}{2\pi} = \frac{1}{8\pi}$$



period $\frac{1}{4}$

18. Determine the general solution for $\cos\left[5\left(x - \frac{\pi}{4}\right)\right] = \frac{\sqrt{2}}{2}$

$$5\left(x - \frac{\pi}{4}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$5\left(x - \frac{\pi}{4}\right) = \begin{cases} \frac{\pi}{4} + 2\pi k, k \in \mathbb{I} \\ \frac{7\pi}{4} + 2\pi k, k \in \mathbb{I} \end{cases}$$

$$x - \frac{\pi}{4} = \begin{cases} \frac{\pi}{20} + \frac{2\pi k}{5}, k \in \mathbb{I} \\ \frac{7\pi}{20} + \frac{2\pi k}{5}, k \in \mathbb{I} \end{cases}$$

$$x = \begin{cases} \frac{3\pi}{10} + \frac{2\pi k}{5}, k \in \mathbb{I} \\ \frac{3\pi}{5} + \frac{2\pi k}{5}, k \in \mathbb{I} \end{cases}$$

What are the solutions between $[0, 2\pi]$? ~ 6.3

Possible Solutions:

$$\checkmark \frac{3\pi}{10}, \left[\frac{3\pi}{10} + \frac{2\pi(1)}{5}\right] = \frac{7\pi}{10} \checkmark$$

$$\left[\frac{3\pi}{10} + \frac{2\pi(2)}{5}\right] = \frac{11\pi}{10}, \checkmark$$

$$\left[\frac{3\pi}{10} + \frac{2\pi(3)}{5}\right] = \frac{3\pi}{2}, \checkmark$$

$$\left[\frac{3\pi}{10} + \frac{2\pi(4)}{5}\right] = \frac{19\pi}{10} \times$$

bigger than 2π