Chapter 7: Exponential Functions

- **graphs**
- **solving equations**
- **word problems**

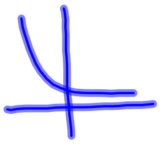
Graphs (Section 7.1 & 7.2):

$$y = c^x$$

- c is the common ratio (can not be 0,1 or a negative)
- if c > 1, growth curve (graph will be increasing)
- if 0< c < 1, decay curve (graph will be decreasing)
- horizontal asymptote y = 0 (the line it approaches but never touches)
- Domain - $x \varepsilon R$
- Range - y > 0
- y-intercept (0,1)

Example 1: State the y-intercept, domain, range, whether it is increasing or decreasing, equation of the horizontal asymptote for each function.

$y = \left(\frac{1}{2}\right)^x$



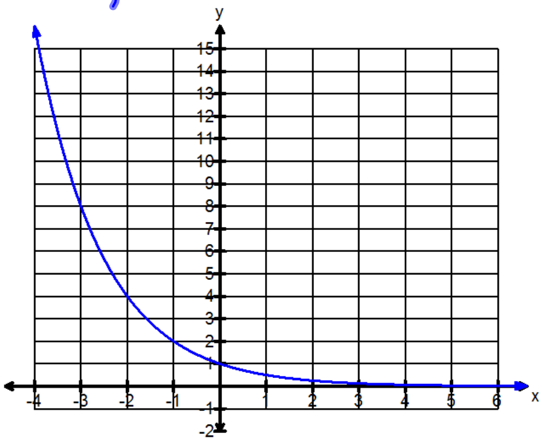
y-int: (0, 1)

Domain: $x \in R$

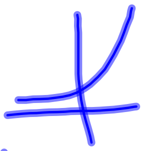
Range: $y > 0$

Decreasing

HA $y = 0$



$y = 4^x$



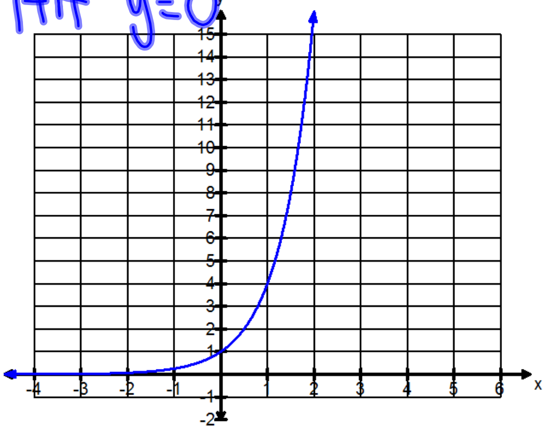
y-int (0, 1)

Domain: $x \in R$

Range: $y > 0$

Increasing

HA $y = 0$



Transformations of Exponential Functions

$$y = ac^{b(x+h)} + k$$

- Vertical Translation
- Horizontal translation
- $|b| = HS$
  if 'b' is negative then graph is reflected in y-axis
- growth/decay
- $|a|$ is the VS
  if 'a' is negative the graph will be reflected in the x-axis

Example 2: Sketch the graph of the function using a mapping rule. State the domain and range, y-intercept and horizontal asymptote.

$$y = -\frac{1}{3}\cdot 2^{x+3} - 1$$

$y = -\frac{1}{3}(2)^{x+3} - 1$

MR: $(x, y) \rightarrow (x-3, -\frac{1}{3}y - 1)$

$y = 2^x$

x	y
-1	1/2
0	1
1	2
2	4
3	8

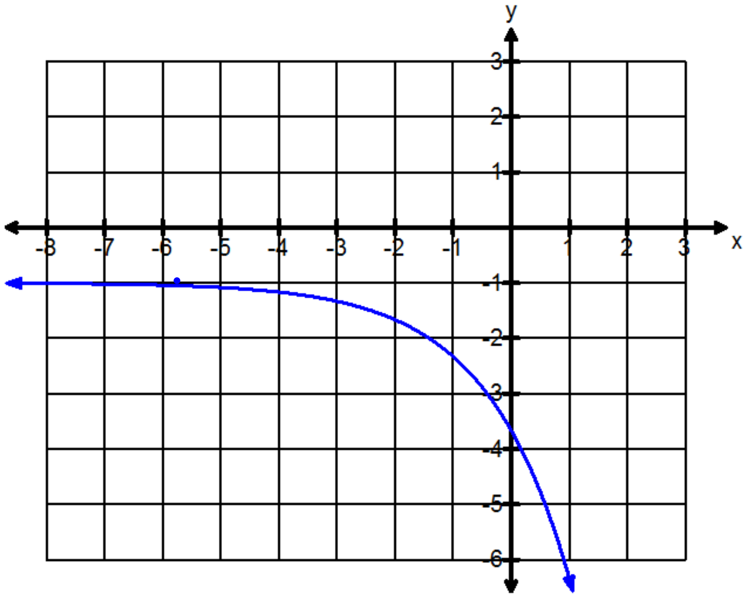
x-3	$-\frac{1}{3}y-1$	
-4	-7/6	(-1.1$\bar{6}$)
-3	-4/3	(-1.3)
-2	-5/3	(-1.6)
-1	-7/3	(-2.3)
0	-11/3	(-3.$\dot{6}$)

Domain: $x \in \mathbb{R}$

Range: $y < -1$

y-int: $(0, -11/3)$

HA: $y = -1$

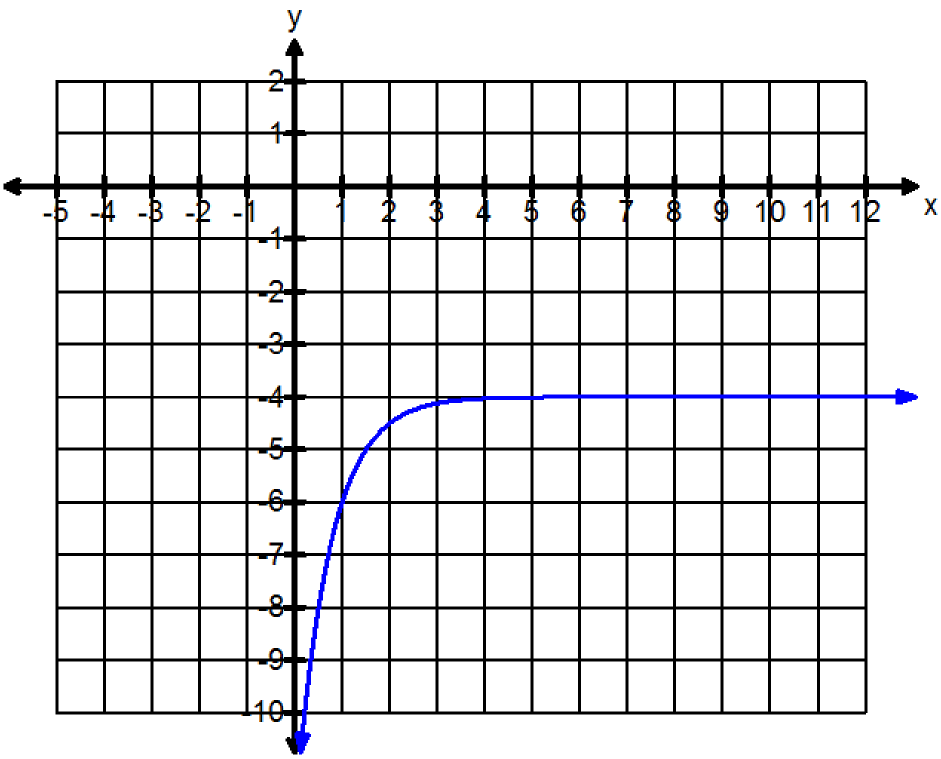

y
x
3
2
1
-1
-2
-3
-4
-5
-6
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3

Example 3: State the domain and range, y-intercept and horizontal asymptote of the function below. State the mapping rule.

$$y = -2 \cdot \left(\frac{1}{2}\right)^{2(x-1)} - 4$$

Domain: $x \in R$
y-int: $(0, -12)$
Range: $y < -4$
Horizontal A: $y = -4$
$(x, y) \rightarrow \left(\frac{1}{2}x + 1, -2y - 4\right)$

y-int
$y = -2\left(\frac{1}{2}\right)^{2(0-1)} - 4$
$= -12$


y
x
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 10 11 12

Assigned Work:
pg 342 #1-5 and pg 354 #1-5(cd), 6, 7

Section 7.3: Solving Exponential Equations

Example 1: Rewrite each expression as a power with a base of 3.

27 9^2 $27^{\frac{1}{3}}(\sqrt[3]{81})^2$

Example 2: Solve each equation

$16^{2x+1} = \left(\frac{1}{2}\right)^{x-3}$

$5\left(\frac{1}{4}\right)^{x} = 80$

$\sqrt[5]{8^{x-1}} = \sqrt[3]{16^{x}}$

Assigned work:
Pg 364 #1, 3, 4, 5 (solve only) and worksheet

Example 3: Solving exponential equations with different bases.

2 choices - trial and error or graphing calculator (this would be a multiple choice!)

Graphing Calculator -
Input the LHS and the RHS on your calculator and see where they intersect.

$$3 = 4^{x+1}$$

#5 and 6 page 364

Word Problems

Example 1: Mary invested $200 in a GIC that paid 6% interest per year. Find the equation of the function that describes the amount of money she has in the fund. How much will she have in 12 years?

$y = 200(1.06)^x$

$y = 200(1.06)^{12}$

$y = 402.44$

$1 - 0.15$

Example 2: John bought a motorcycle for $3000. It depreciates by 15% of it's original value each year. Write the function that represents this situation and use it to determine how much the motorcycle is worth after 5 years.

$y = 3000(0.85)^x$

$= 3000(0.85)^5$

$= \$1331.12$

Example 3: An SUV that is originally worth $50,000 depreciates at a rate of 29.5% every two years. Find a function for the depreciation of the SUV, and how much will it be worth after 3 years?

$y = 50\,000(0.705)^{x/2}$

$y = 50\,000(0.705)^{3/2}$

$y = \$29597.41$

Example 4: A radioactive substance has a half-life of 17 days. Write a function to model this situation and use it to determine the time it will take for 400 g of this substance to decay to 100 g.

$y = 400\left(\frac{1}{2}\right)^{x/17}$

$\frac{100}{400} = \frac{400\left(\frac{1}{2}\right)^{x/17}}{400}$

$\frac{1}{4} = \left(\frac{1}{2}\right)^{x/17}$

$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{x/17}$

$2 = \frac{x}{17}$

$x = 34$

Example 5: An element has a half-life of 120 years. If its initial mass is 42 grams, algebraically determine how long it will take to decrease to 10.5 grams.

$y = 42\left(\frac{1}{2}\right)^{x/120}$

$10.5 = 42\left(\frac{1}{2}\right)^{x/120}$

$\left(\frac{1}{4}\right) = \left(\frac{1}{2}\right)^{x/120}$

$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{x/120}$

$2 = x/120$

$x = 240$

Example 6: Hot chocolate cools exponentially over time after its brought into a stadium. Its cooling is described by the function $T = 45(0.95)^m + 5$.

a) What is the temperature as soon as you bring it into the stadium?

50° (y-int)

b) What is the air temperature in the stadium?

5° (HA)

Example 7: A chemical substance has a half-life of 65 minutes. When will there be 1/64th of the original amount?

$$y = a\left(\frac{1}{2}\right)^{x/65}$$

$$\frac{1}{64}a = a\left(\frac{1}{2}\right)^{x/65}$$

$$\frac{1}{64} = \left(\frac{1}{2}\right)^{x/65}$$

$$\left(\frac{1}{2}\right)^{6} = \left(\frac{1}{2}\right)^{x/65}$$

$$6 = \frac{x}{65}$$

$$\boxed{x = 390}$$

Worksheet

Compound Interest Word Problems

$$A = A_o(1+r)^n$$

A_o = initial amount invested
r = rate (annual interest rate/compounding periods annually)
n = total number of compounding periods

Example 7: You deposit $2000 in an account that earns 5% annual interest. Find the balance after 2 year if the interest is compounded with the given frequency.

A) Annually

$$A = 2000(1+0.05)^n$$
$$= 2000(1+0.05)^2$$
$$= \$2205.00$$

B) Quarterly

$$A = 2000\left(1+\frac{0.05}{4}\right)^n$$
$$= 2000\left(1+\frac{0.05}{4}\right)^8$$
$$= \$2208.97$$

C) Monthly

$$A = 2000\left(1+\frac{0.05}{12}\right)^{24}$$
$$= \$2209.88$$

Complete questions 11a,b and 13 ab pg 365