Chapter 2: Radical Functions

Section 2.1: Radical Functions and Transformations

Example 1: pg 63 in text

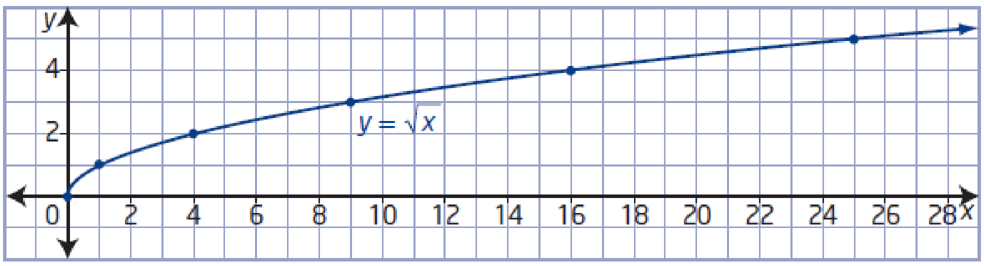
Use a table of values to sketch the graph of each function. Then, state the domain and range of each function.

a) $y = \sqrt{x}$ **b)** $y = \sqrt{x - 2}$ **c)** $y = \sqrt{x} - 3$

For the function $y = \sqrt{x}$, the radicand x must be greater than or equal to zero, $x \geq 0$.

a) $y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4
25	5



Domain: $\{x \mid x \geq 0, x \in R\}$

Range: $\{y \mid y \geq 0, y \in R\}$

b)

$x - 2 \geq 0$

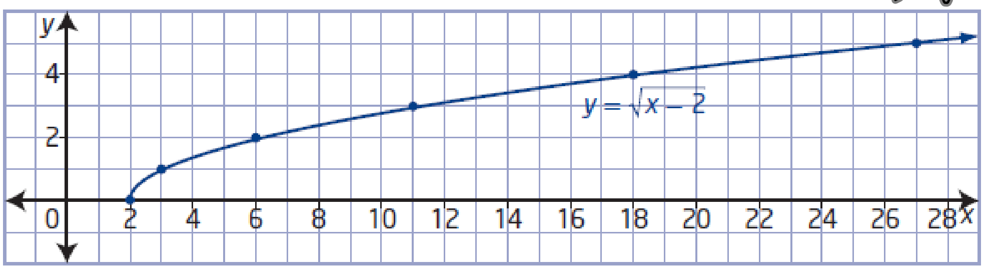
$x \geq 2$

b) $y = \sqrt{x-2}$

x	y
2	0
3	1
6	2
11	3
18	4
27	5

$\{x \mid x \geq 2, x \in R\}$

$\{y \mid y \geq 0, y \in R\}$

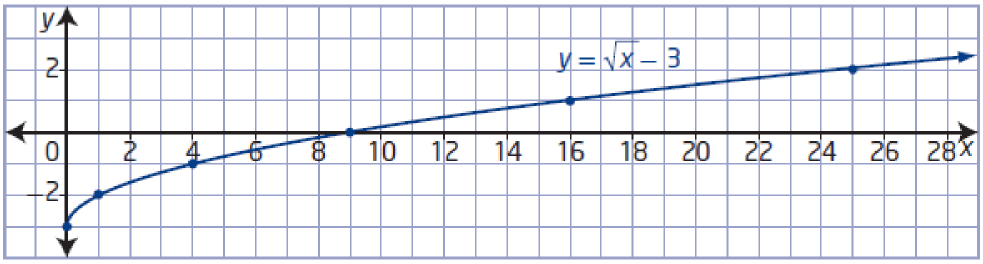


The domain is $\{x \mid x \geq 2, x \in R\}$. The range is $\{y \mid y \geq 0, y \in R\}$.

c)

$x \geq 0$

x	y
0	−3
1	−2
4	−1
9	0
16	1
25	2



The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -3, y \in \mathrm{R}\}$.

Graphing Radical Functions Using Transformations

You can graph a radical function of the form $y = a\sqrt{b(x - h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a, b, h, and k. The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter a results in a vertical stretch of the graph of $y = \sqrt{x}$ by a factor of $|a|$. If $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis.
- Parameter b results in a horizontal stretch of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{|b|}$. If $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis.
- Parameter h determines the horizontal translation. If $h > 0$, the graph of $y = \sqrt{x}$ is translated to the right h units. If $h < 0$, the graph is translated to the left $|h|$ units.
- Parameter k determines the vertical translation. If $k > 0$, the graph of $y = \sqrt{x}$ is translated up k units. If $k < 0$, the graph is translated down $|k|$ units.

pg 72 #1,2,3,4

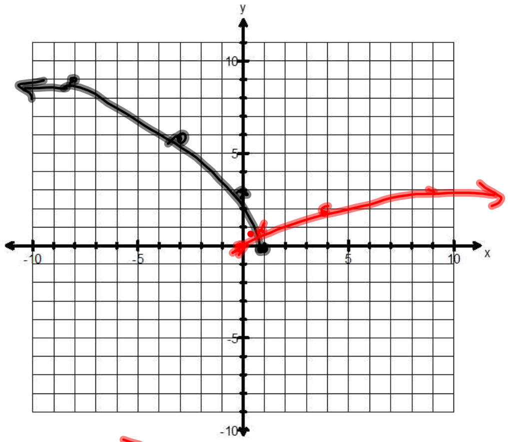
Example 2: pg 65

Sketch the graph of the function using transformations. State the domain and range.

$$y = 3\sqrt{-(x-1)}$$

$y = \sqrt{x}$

Mapping Rule: $(x, y) \rightarrow (-x+1, 3y)$



Original	*New*
(0,0)	(1, 0)
(1,1)	(0, 3)
(4, 2)	(-3, 6)
(9,3)	(-8, 9)

$D = \{x \mid x \leq 1, x \in R\}$

$R = \{y \mid y \geq 0, y \in R\}$

worksheet example 1-5 and text pg 73 #5

Section 2.2: Square Root of a Function

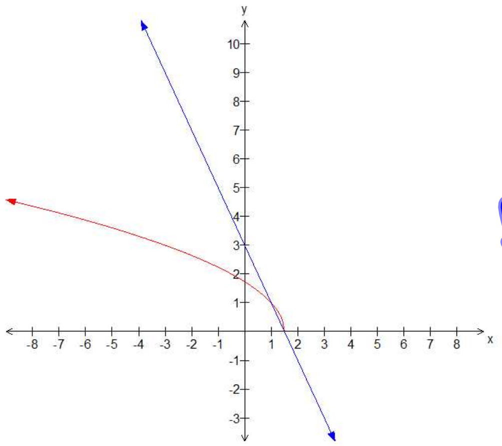
Example 1: pg 80

Linear function vs. Square Root

a) Given f(x) = 3 - 2x, graph the functions y = f(x) and $y=\sqrt{f(x)}$

a)

x	y = 3 - 2x	$y=\sqrt{3-2x}$
-2	7	$\sqrt{7}$
-1	5	$\sqrt{5}$
0	3	$\sqrt{3}$
1	1	1
1.5	0	0



invariant points occur at (1,1) and (1.5, 0)

$f(x) = 3-2x$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

$y = \sqrt{3-2x}$

Domain: $\{x \mid x \leq 1.5, x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

Note: The ***domain*** of $y = \sqrt{f(x)}$ consists only those values in the domain of f(x) for which f(x) $\geq$ 0

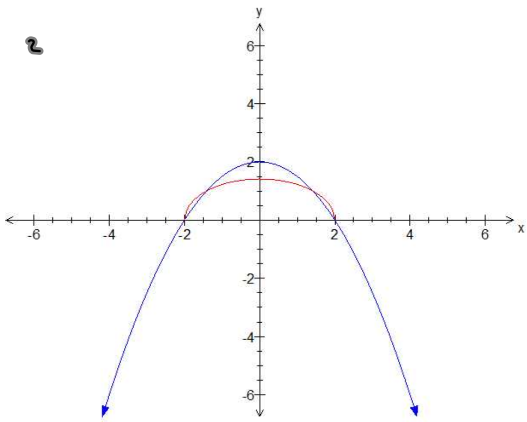
The ***range*** of $y = \sqrt{f(x)}$ consists of the square roots of the values in the range of y = f(x) for which $\sqrt{f(x)}$ is defined.

Example 2: What are the domain and range for each function?

$$y = 2 - 0.5x^2 \qquad y = \sqrt{2 - 0.5x^2}$$

Locate any intercepts and the maximum value or minimum value to determine the domain and range of each function.

function	$y = 2 - 0.5x^2$	$y = \sqrt{2 - 0.5x^2}$
x-intercepts	-2 and 2	-2, 2
y - intercept	2	$\sqrt{2}$
maximum value	2	$\sqrt{2}$
minimum value	none	0



original

$\{x \,/\, x \;\varepsilon\; R\}$

$y \,/\, y \le 2, y \;\varepsilon\; R\}$

NEW

$\{x \,/\, -2 \le x \le 2, x \;\varepsilon\; R\}$

$\{y \,/\, 0 \le y \le \sqrt{2}, y \;\varepsilon\; R\}$

Ex 2 b pg 82

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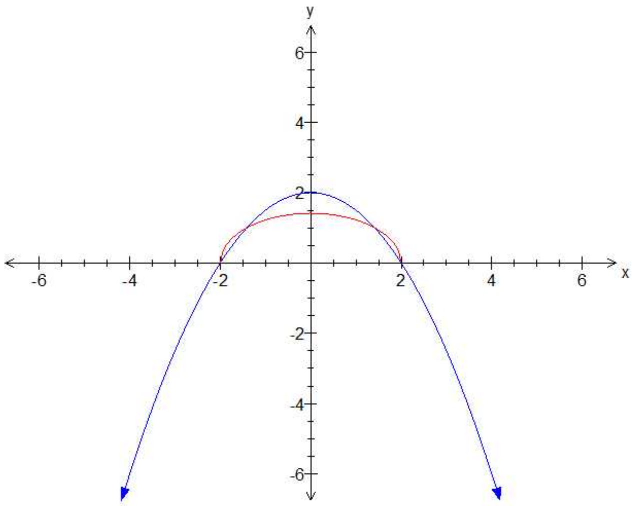
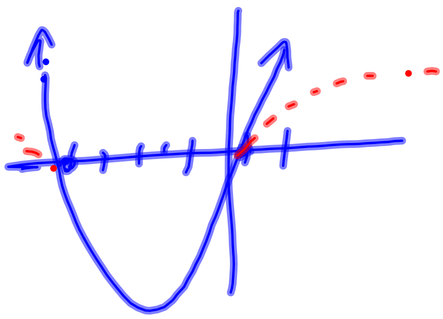
#1, 2, 3, 5, 6, 7


y
6
4
2
-2
-4
-6
x
-6
-4
-2
2
4
6

Example 2B: Determine the domain and range of the following functions.

$y = \sqrt{x^2 + 4x - 5}$



$y = x^2 + 4x - 5$

$0 = (x+5)(x-1)$

$x = -5$ or $x = 1$

$D = \{x \mid x \geq 1 \text{ or } x \leq -5, x \in R\}$

$R = \{y \mid y \geq 0, y \in R\}$

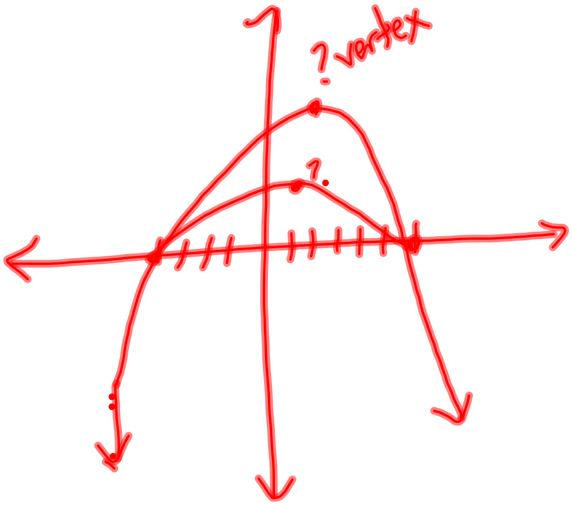
$y = \sqrt{-\frac{1}{2}x^2 + x + 12}$

$y = -\frac{1}{2}x^2 + x + 12$

$y = -\frac{1}{2}(x^2 - 2x - 24)$

$y = -\frac{1}{2}(x-6)(x+4)$

$x = 6$ or $x = -4$



Vertex of Original

$x = \frac{-b}{2a} = \frac{-1}{2(-1/2)} = 1$

$y = -\frac{1}{2}x^2 + x + 12$

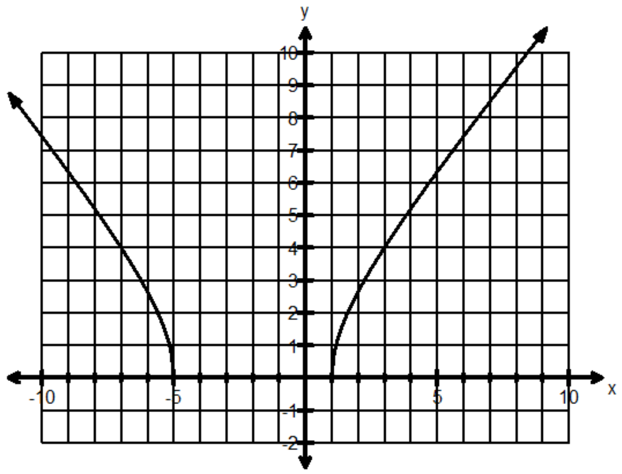
$= -\frac{1}{2}(1)^2 + (1) + 12$

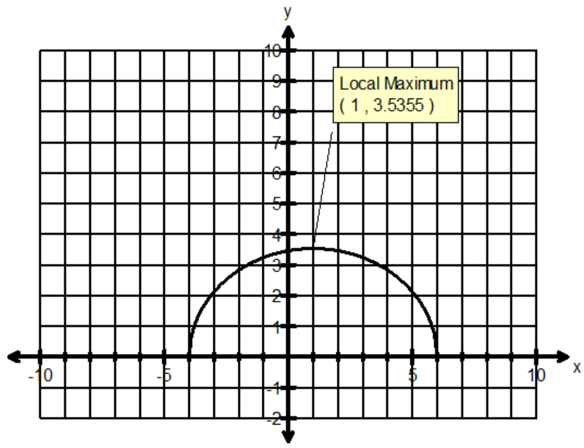
$= 12.5$

$(1, 12.5)$

$D = \{x \mid -4 \le x \le 6, x \in R\}$

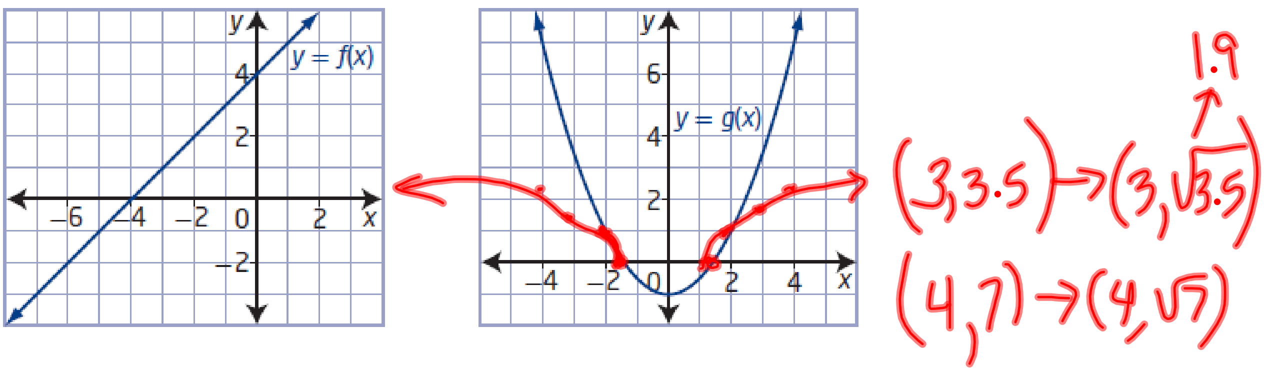
$R = \{y \mid 0 \le y \le \sqrt{12.5}, y \in R\}$


y
x
-10
-5
5
10
10
9
8
7
6
5
4
3
2
1
-1
-2


y
x
Local Maximum
(1 , 3.5355)
-10
-5
5
10
10
9
8
7
6
5
4
3
2
1
-1
-2

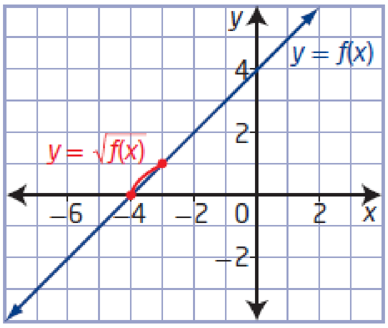
Example 3:

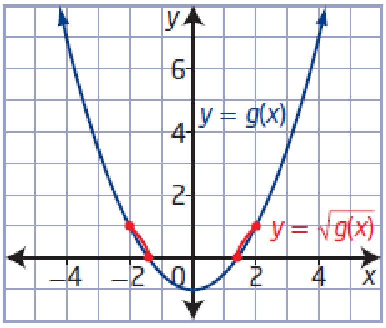
Using the graphs of $y = f(x)$ and $y = g(x)$, sketch the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.



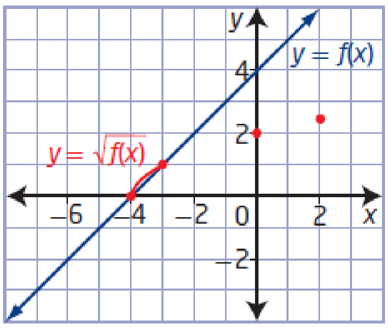
Step 1: Locate invariant points on $y = f(x)$ and $y = g(x)$. When graphing the square root of a function, invariant points occur at $y = 0$ and $y = 1$.

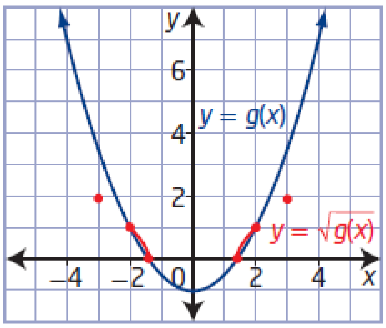
Step 2: Draw the portion of each graph between the invariant points for values of $y = f(x)$ and $y = g(x)$ that are positive but less than 1. Sketch a smooth curve *above* those of $y = f(x)$ and $y = g(x)$ in these intervals.



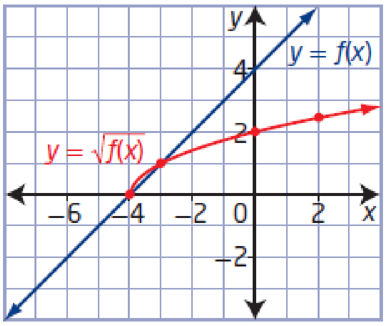


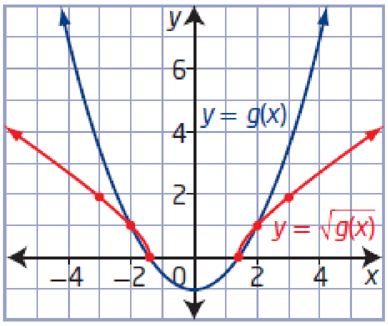
Step 3: Locate other key points on $y = f(x)$ and $y = g(x)$ where the values are greater than 1. Transform these points to locate image points on the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.





Step 4: Sketch smooth curves between the image points; they will be below those of $y = f(x)$ and $y = g(x)$ in the remaining intervals. Recall that graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$ do not exist in intervals where $y = f(x)$ and $y = g(x)$ are negative (below the x-axis).





8,11,13,17,

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Section 2.3: Solving Radical Equations Graphically

Example 2: Pg 92 Solve the equations graphically and algebraically

$\sqrt{x+5} = x+3$

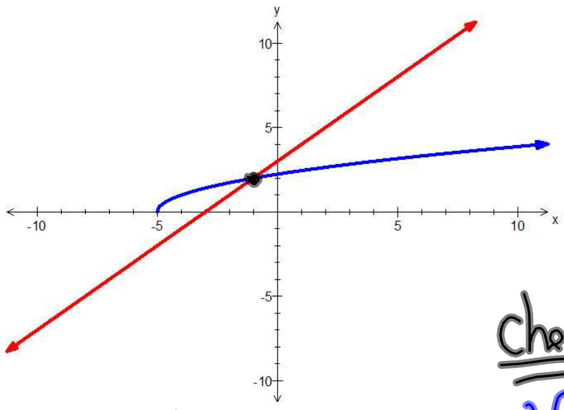
$(\sqrt{x+5})^2 = (x+3)^2$

$x+5 = x^2+6x+9$

$0 = x^2+5x+4$

$0 = (x+4)(x+1)$

$x=-4, x=-1$



Check:

$\sqrt{x+5} = x+3$

$\sqrt{-4+5} \stackrel{?}{=} -4+3$

$\sqrt{1} \stackrel{?}{=} -1$

$1 \neq -1$

$(-1, 2)$

Check:

$\sqrt{x+5} = x+3$

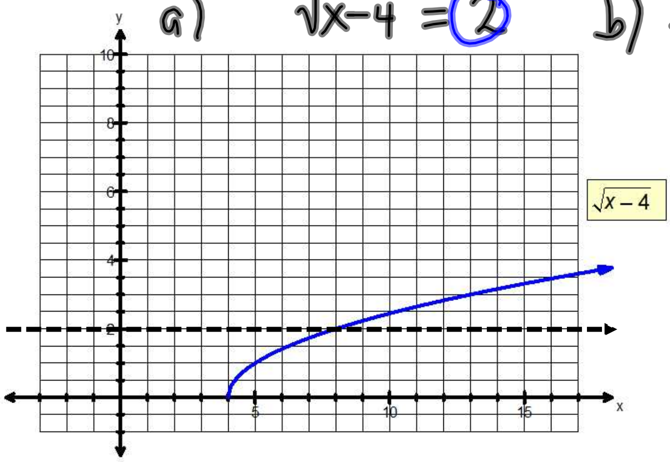
$\sqrt{-1+5} \stackrel{?}{=} -1+3$

$\sqrt{4} \stackrel{?}{=} 2$

$2 \stackrel{\checkmark}{=} 2$

Example 2B: Solve graphically & algebraically

a) $\sqrt{x-4} = 2$ b) $\sqrt{x-4} = -3$



Worksheet

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