Chapter 1: Function Transformations

Section 1.1: Horizontal and Vertical Translations

transformation: alters the equation and any combination of the location, shape, and orientation of the graph

mapping: the relationship between a set of points of an original graph and the transformed graph

translation: movement of a graph up, down, right or left

Example 1: ***pg 8***

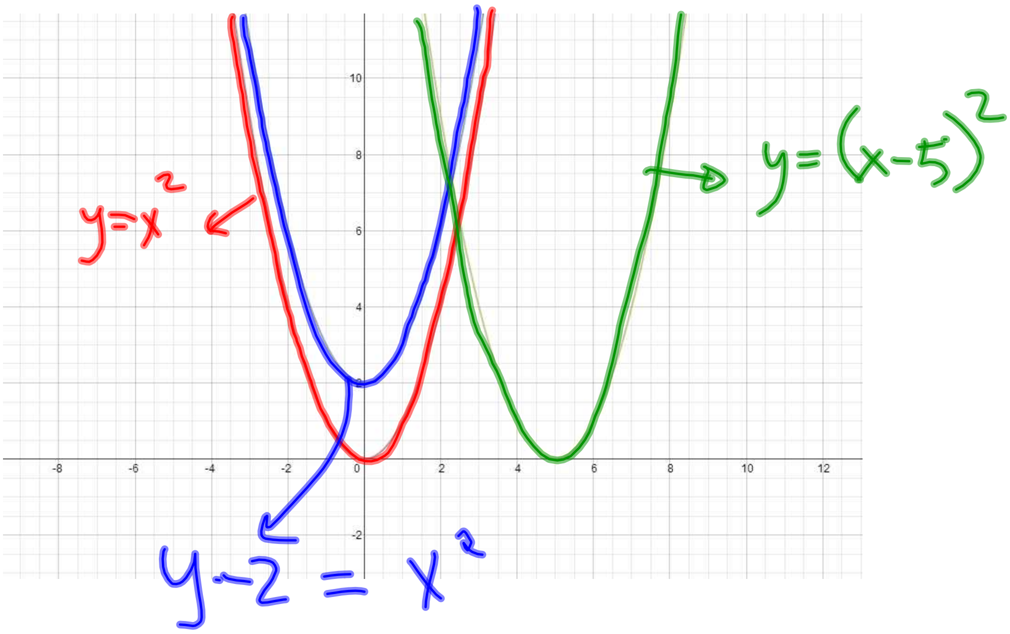
a) Graph the functions $y = x^2$, $y - 2 = x^2$ and $y = (x - 5)^2$ on the same set of coordinate axes.

b) Describe how the graphs of $y - 2 = x^2$ and $y = (x - 5)^2$ compare to the graph of $y = x^2$

up 2 right 5

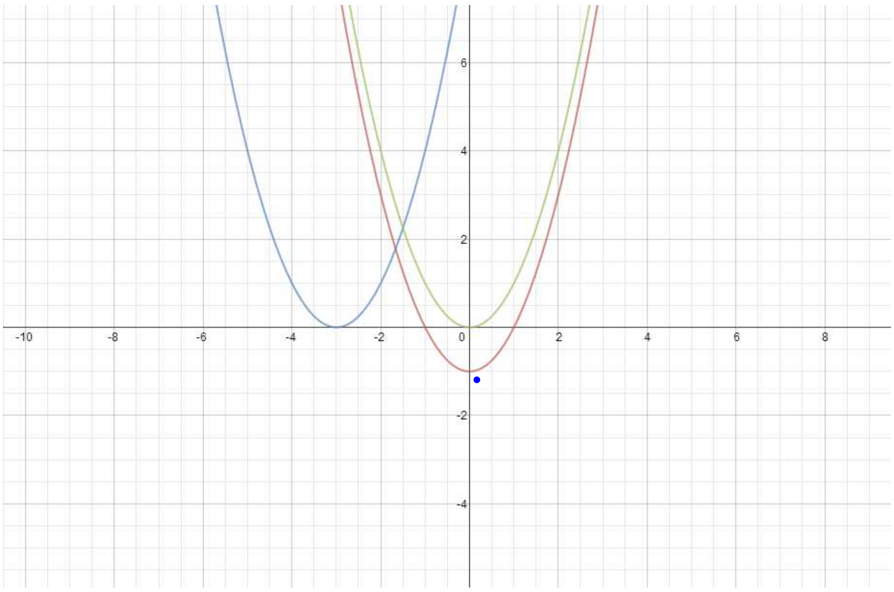
a)

$y = x^2$	$y - 2 = x^2$	$y = (x - 5)^2$
(-2, 4)	(-2, 6)	(2, 9)
(-1, 1)	(-1, 3)	(3, 4)
(0, 0)	(0, 2)	(4, 1)
(1, 1)	(1, 3)	(5, 0)
(2, 4)	(2, 6)	(6, 1)


$y = x^2$
$y = (x-5)^2$
$y - 2 = x^2$
10
8
6
4
-2
-8
-6
-4
-2
0
2
4
6
8
10
12

Your Turn pg 9

How do the graphs of $y + 1 = x^2$ and $y = (x + 3)^2$ compare to the graph of $y = x^2$?



Summary: A transformed function $y-k=f(x-h)$ is translated *k* units vertically and *h* units horizontally

$y+5=f(x+7)$

Ex: $y+4=f(x-7)$

mapping: $(x,y)\to(x+7,y-4)$

Ex: $\cancel{f}^{y}(x)=f(x+7)-5$

mapping:

$(x,y)\to(x-7,y-5)$

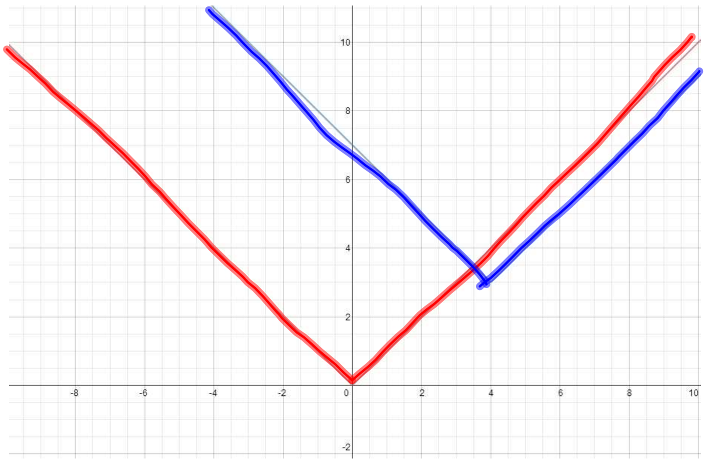
What is happening in the graph?

Example 2: Sketch the graph of $y = |x-4| + 3$ $y - 3 = |x-4|$

***mapping as compared to y = $|x|$

(x, y) ⟶ (x + 4, y + 3)

y = |x|	(x + 4, y + 3)
(-2, 2)	(2, 5)
(-1, 1)	(3, 4)
(0, 0)	(4, 3)
(1, 1)	(5, 4)
(2, 2)	(6, 5)
	(,)


10
8
6
4
2
-2
-8
-6
-4
-2
0
2
4
6
8
10

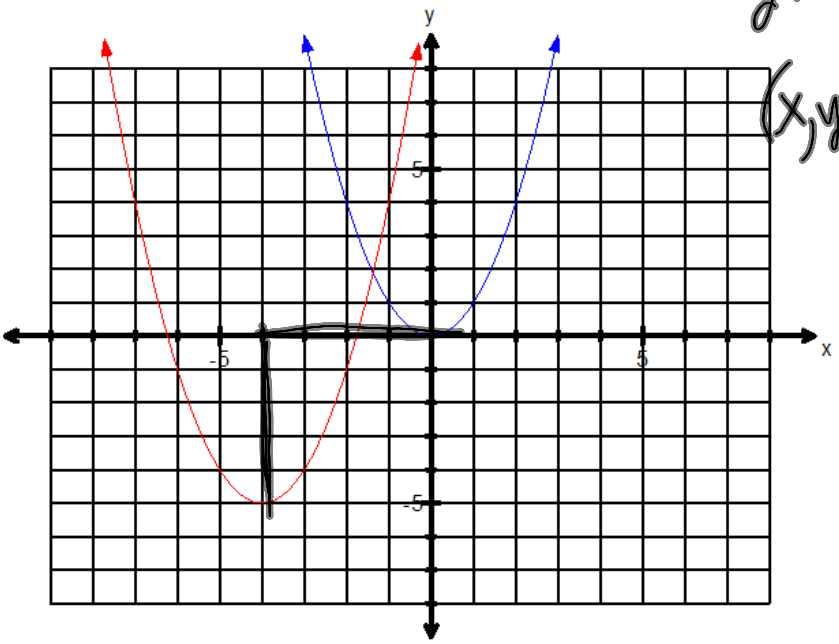
pg 12 #1-4, 8, 10

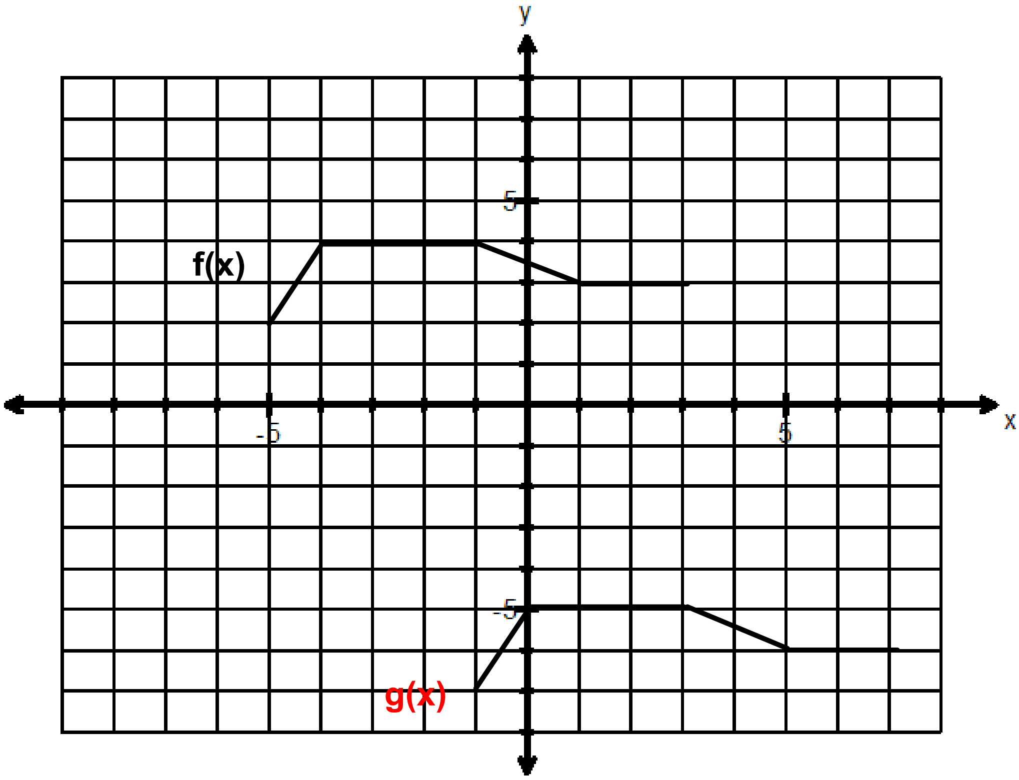
Example 3: pg 10

Determine the equation of the translated function in the form y - k = f(x - h)

$y+5 = f(x+4)$

$(x,y) \to (x-4, y-5)$





#8, 10 pg 13

#11 pg 14

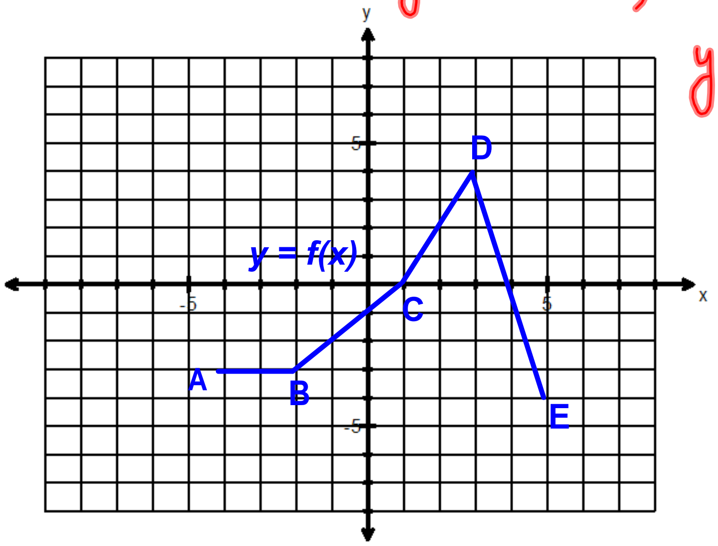
Section 1.2: Reflections and Stretches

Reflections

Example 1: Pg 18

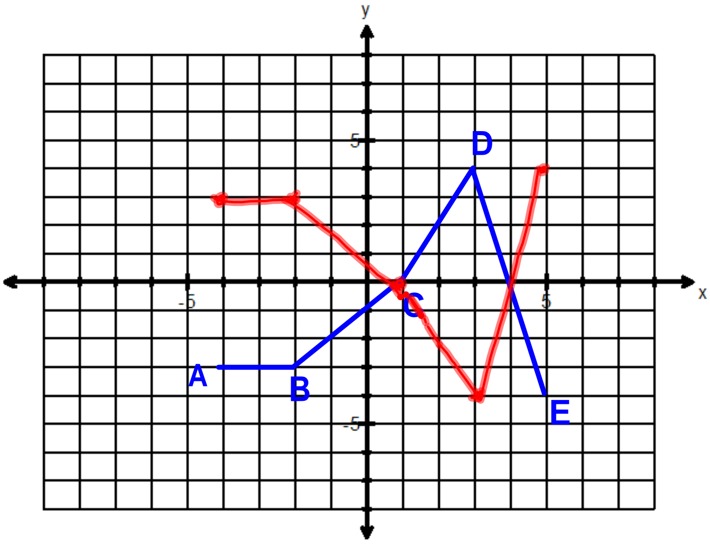
$y = f(x)$ $y = f(-x)$

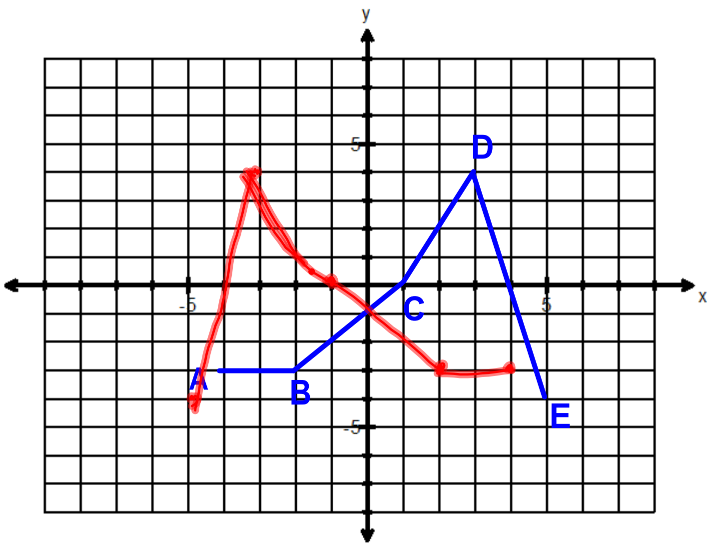
$y = -f(x)$



Graph of *y = -f(x)*

y = f(x)	*y = -f(x)*	*y = f (-x)*
(-4,-3)	(-4, 3)	(4, -3)
(-2,-3)	(-2, 3)	(2, -3)
(1, 0)	(1, 0)	(-1, 0)
(3, 4)	(3, -4)	(-3, 4)
(5,-4)	(5, 4)	(-5, -4)


y
5
-5
5
-5
x
A
B
C
D
E


y
x
5
-5
5
-5
A
B
C
D
E

Summary:

$y = -f(x)$ produces a reflection in the x-axis
mapping rule: $(x, y) \rightarrow (x, -y)$

$y=f(-x)$ produces a reflection in they-axis
mapping rule: $(x, y) \longrightarrow (-x, y)$

Note: Invariant Points!!

Vertical and Horizontal Stretches

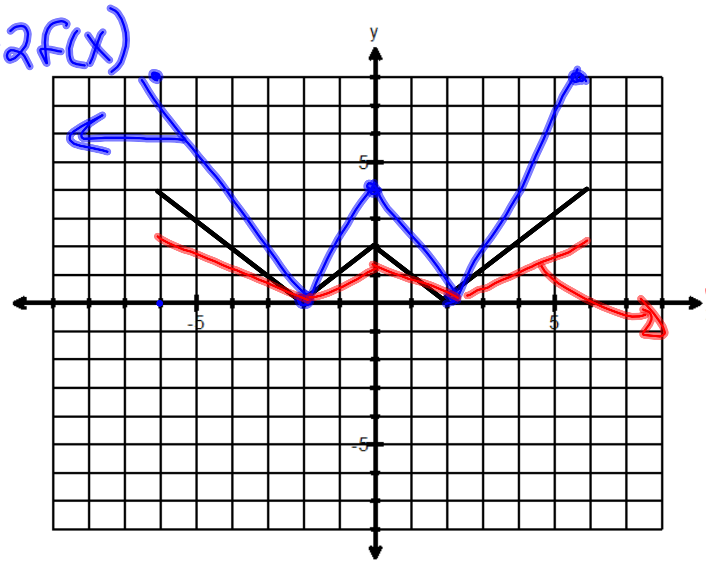
Example 2: pg 21

Compare *y* = *f*(*x*) to *g*(*x*) = 2*f*(*x*)

$h(x) = \frac{1}{2}f(x)$

$(x,y) \to (x, 2y)$

$g(x)=2f(x)$



$y = \frac{1}{2}f(x)$

Mapping Rule:(x, y) → (x, 2y)

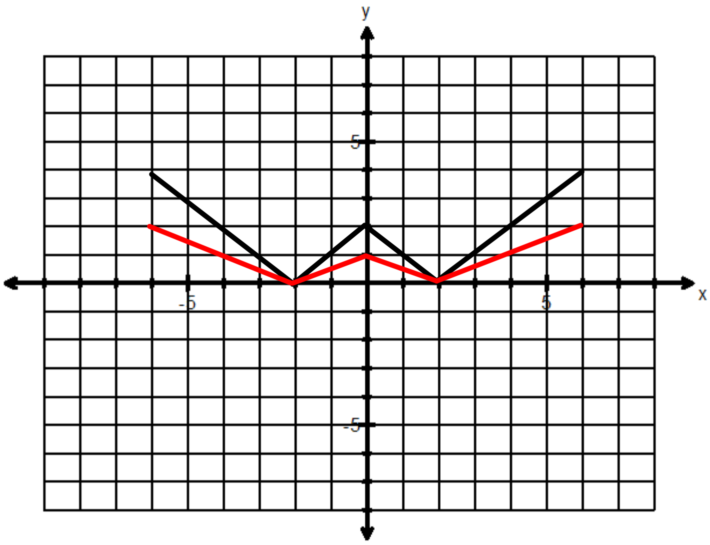
vertical stretch of the graph y = f(x) about the x-axis by a factor of 2

invariant point (-2,0) and (2,0)

Domain $\{x \mid -6 \le x \le 6, x \in R\}$

Range $\{y \mid 0 \le y \le 8, y \in R\}$

Compare $y = f(x)$ to $r(x) = 1/2\ f(x)$



(x, y) (x, 1/2 y)

invariant points (-2,0) and (2,0)
Domain
Range

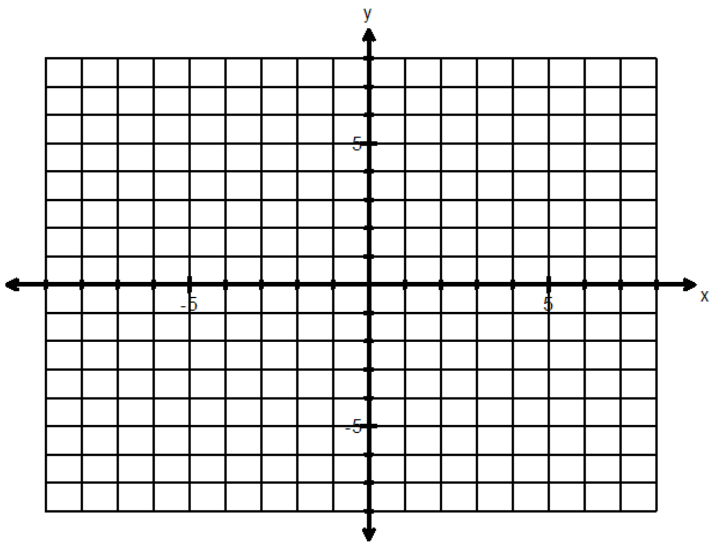
pg 28 #1- 6

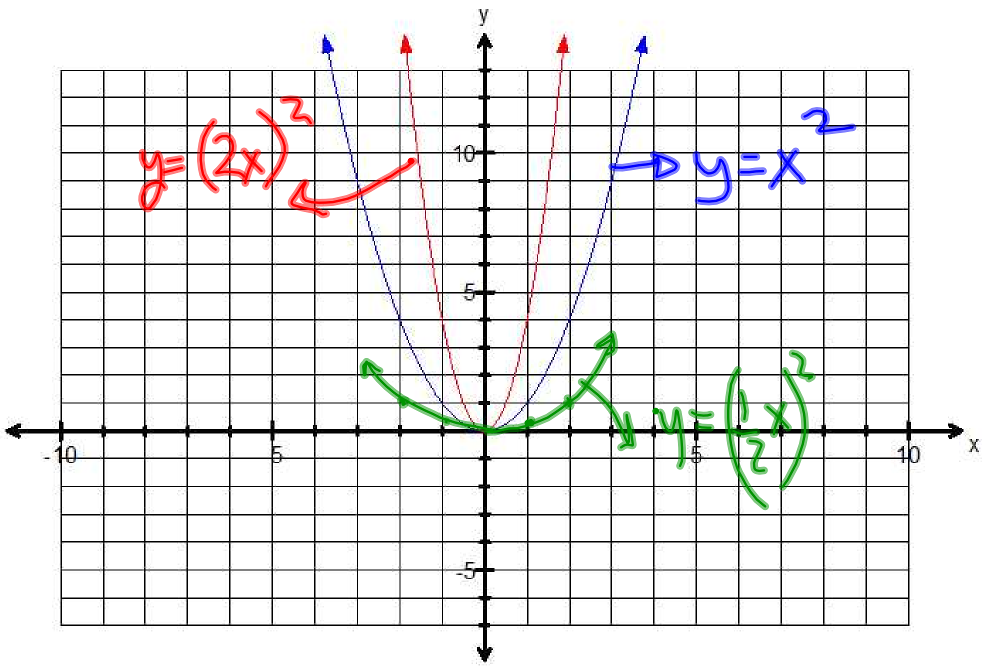
omit #5

***Horizontal Stretches:**

Compare the graphs of $y = x^2$, $y = (2x)^2$ and $y = (1/2x)^2$

$y = x^2$	$y = (2x)^2$	$y = (1/2x)^2$
(-2, 4)	(-2, 16)	(-2, 1
(-1, 1)	(-1, 4)	(-1, 1/4)
(0, 0)	(0, 0)	(0, 0)
(1, 1)	(1, 4)	(1, 1/4)
(2, 4)	(2, 16)	(2, 1)


y
5
-5
5
-5
x


y
x
10
5
-5
-10
-5
5
10
y=(2x)^2
y=x^2
y=(1/2 x)^2

Horizontal Stretches

Summary:

- *y* = *f(bx)* means a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$
- If b is less than 0, the graph is also reflected in the y-axis
- mapping rule: $(x,y) \rightarrow (\frac{x}{b}, y)$

Example: The graph of y = f(3x) is a horizontal stretch by a factor of 1/3, of the graph of y = f(x)

$$(x,y) \rightarrow (\frac{1}{3}x, y)$$

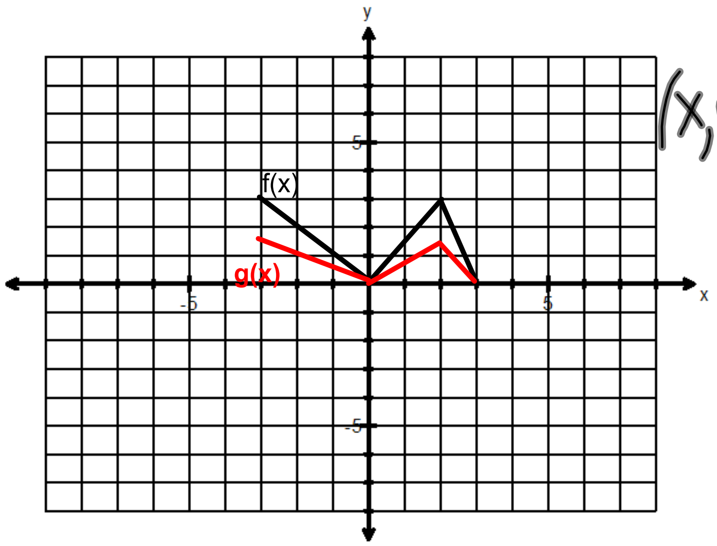
Transformations from a graph

compare y = f(x) and g(x)

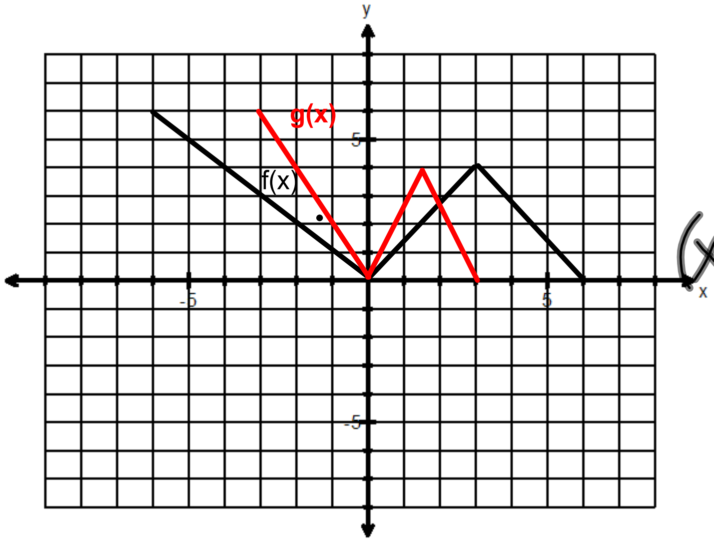
$g(x) = \frac{1}{2} f(x)$

$(x, y) \rightarrow (x, \frac{1}{2} y)$

Reflection?
Vertical Stretch?
Horizontal Stretch?



$g(x) = f(2x)$

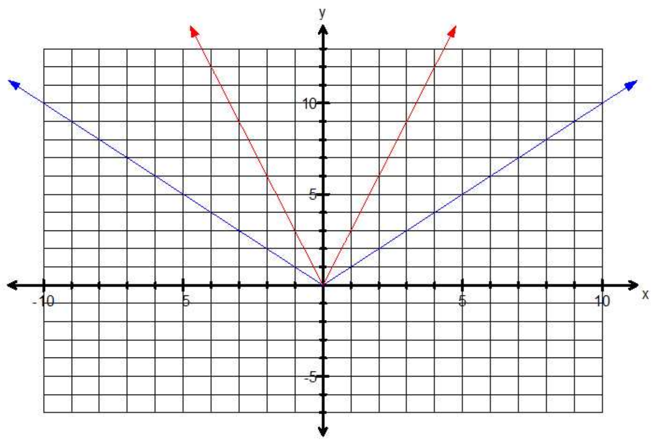


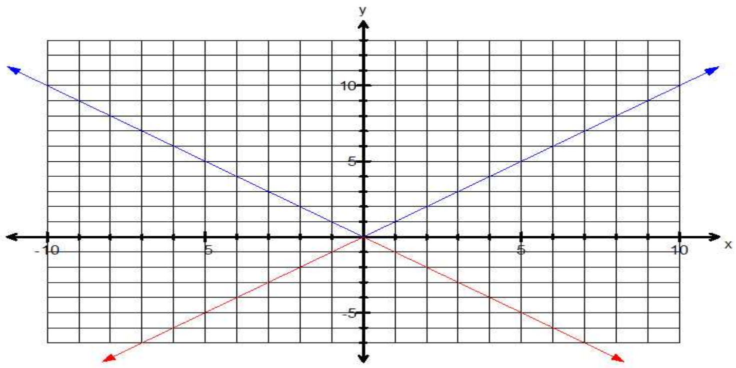
Reflection?
Vertical Stretch?
Horizontal Stretch?

$(x, y) \rightarrow (\frac{1}{2}x, y)$

Example 4: pg 25

Write the equation of the transformed graph.




y
10
5
-5
-10
5
5
10
x

Section 1.3: Combining Transformations

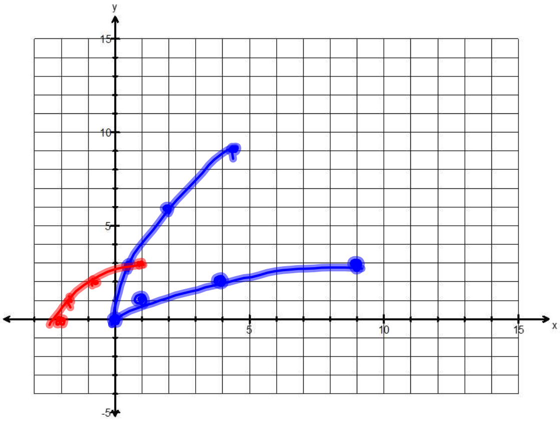
Multiple transformations can be applied to a function using the general transformations model

$$y = af\left(b\left(x-h\right)\right)+k \qquad \textit{or} \qquad y-k = af\left(b\left(x-h\right)\right)$$

To accurately sketch the graph of the transformed function the stretches and reflections should occur before the translations

Recommended Sequence (pg 34 in text)

Example 1: Describe the combination of transformations that must be applied to *y* = *f(x)* to obtained the transformed function. State the mapping rule and sketch the graph.



a) *y = 3f(2x)*

Horizontal stretch of 1/2
Vertical Stretch of 3

$(x, y) \rightarrow (\frac{1}{2}x, 3y)$

x	y
0	0
1	1
4	2
9	3

$\frac{1}{2}x$	$3y$
0	0
0.5	3
2	6
4.5	9

* b) *y = f(3x+6)*

$y = f(3(x+2))$

$(x, y) \rightarrow (\frac{1}{3}x - 2, y)$

$\frac{1}{3}x - 2$	y
-2	0
$-5/3$	1
$-2/3$	2
1	3

horizontal stretch of 1/3
shifted 2 left

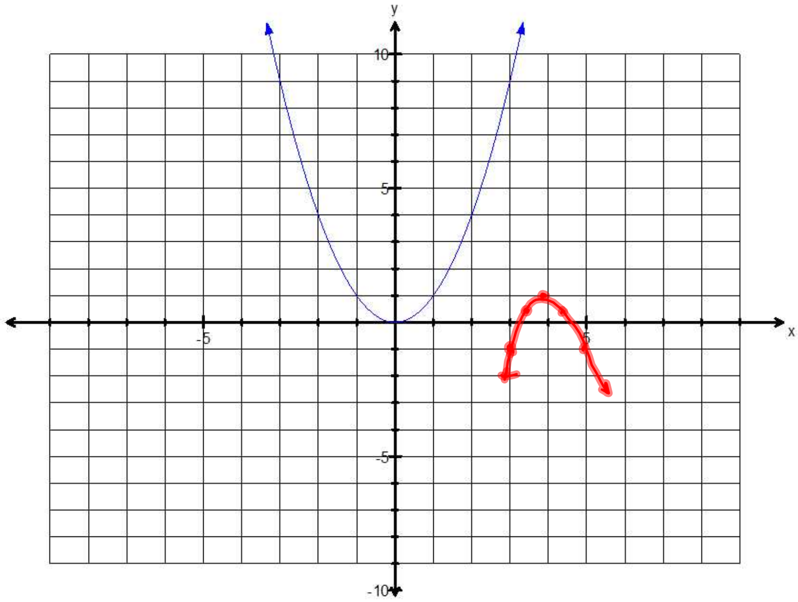
Example 2: Show the combination of transformations that should be applied to the graph of f(x) = x^2 in order to obtain the graph of the transformed function

$$g(x) = -\frac{1}{2} f(2(x-4)) + 1$$

$$(x, y) \rightarrow \left(\frac{1}{2}x + 4, -\frac{1}{2}y + 1\right)$$

x	y
-2	4
-1	1
0	0
1	1
2	4

$\frac{1}{2}x+4$	$-\frac{1}{2}y+1$
3	-1
3.5	0.5
4	1
4.5	0.5
5	-1

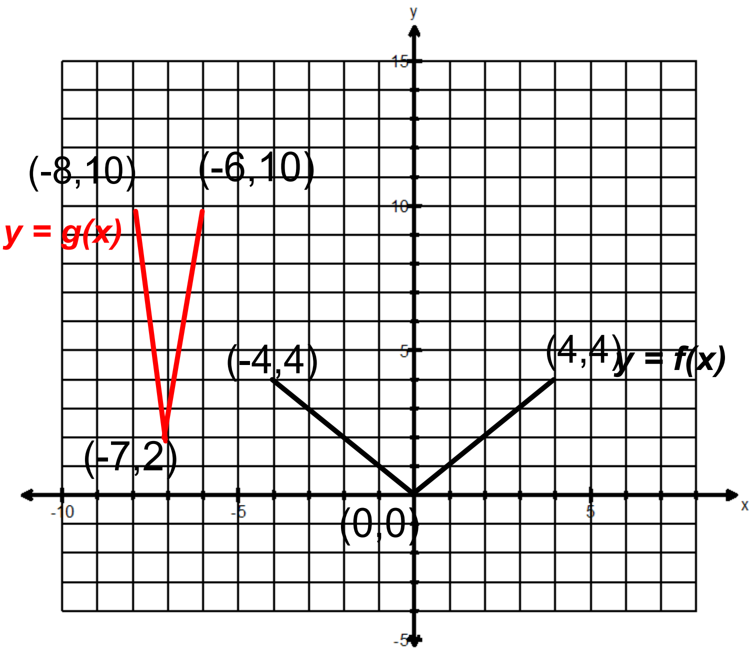


Write the corresponding equation for g(x).

$$g(x) = -\frac{1}{2}(2(x-4))^2 + 1$$

Summary!!! Invariant pts vertical stretch, reflects etc.....

Example 3: Write the equation of a transformed function graph.



$y = 2f(4(x+7)) + 2$

Stretches
'original' domain - 8
'new' domain - 2 therefore horizontal stretch of 1/4

'original' range - 4
'new' range - 8 therefore vertical stretch of 2

$y = f(4x+8)$
$= f(4(x+2))$

translations -use the point (0, 0) as our reference point as it is not affected by a stretch or reflection
horizontal - -7 and vertical - 2

#2-10, 12, 15 pg 38-41