

Unit: Logarithmic Functions

Section A: Selected Response: Place the letter of your response in the space at the right.
(15 marks)

1. What is the exact value of x : $3 = 2^{x+1}$

A) $\log\left(\frac{3}{2}\right) - 1$

B) $\frac{\log 3}{\log 2} - 1$

C) $\log\left(\frac{3}{2}\right) + 1$

D) $\frac{\log 3}{\log 2} + 1$

$\log 3 = \log 2^{x+1}$
 $\log 3 = (x+1) \log 2$
 $\frac{\log 3}{\log 2} = x+1$

1. B

2. Solve for x : $\log_4(3x) + \log_4(x-2) = \log_4 24$

A) $x = -4$

B) $x = 4$

C) $x = -2$

D) $x = 2$

$\log 3x(x-2) = \log 24$
 $3x^2 - 6x = 24$
 $3x^2 - 6x - 24 = 0$
 $3(x^2 - 2x - 8) = 0$
 $(x-4)(x+2) = 0$

2. B

3. What is the domain of $y = -\log_5(6-x)$?

A) $x > -6$

B) $x < -6$

C) $x > 6$

D) $x < 6$

$6-x > 0$
 $6 > x$

3. D

4. What is the equivalent logarithmic form of $3^y = x+1$?

A) $y = \log_3(x-1)$

B) $y = \log(3x+1)$

C) $y = \log_3(x+1)$

D) $y = \log(3x-1)$

$\log_3(x+1) = y$

4. C

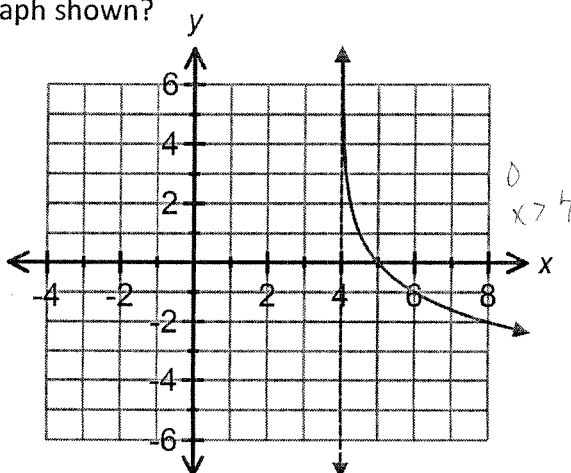
5. What is the equation of the graph shown?

A) $y = \log_2(x-4)$

B) $y = -\log_2(x-4)$

C) $y = -\log_2(x+4)$

D) $y = \log_2(x+4)$



5. B

6. Solve for x: $\log_2(\log_x 64) = 1$

$2^1 = \log_x 64$

6. A

A) $x = 8$

B) $x = 4$

$x^2 = 64$

C) $x = 16$

D) $x = 32$

7. Which expression is equivalent to $\log \frac{A^3}{\sqrt{BC^4}}$?

7. D

A) $3 \log A - \frac{1}{2} \log B + 4 \log C$

$3 \log A - \frac{1}{2} \log B - 4 \log C$

B) $3 \log A - \frac{1}{2} \log B + 2 \log C$

C) $3 \log A - \frac{1}{2} \log B - 2 \log C$

D) $3 \log A - \frac{1}{2} \log B - 4 \log C$

8. Solve for x: $\log_6(5x + 2) = \frac{1}{2} \log_6 64 + \log_6 3$

8. B

A) $x = \frac{94}{5}$

B) $x = \frac{22}{5}$

~~$\log_6(5x+2) = \log_6(8)(3)$~~

C) $x = \frac{26}{5}$

D) $x = \frac{9}{5}$

$5x + 2 = 24$

$5x = 22$

$x = \frac{22}{5}$

9. Write as a single logarithm $3[\log A + \log B] - \log C$.

9. C

A) $\log \frac{AB}{C^3}$

$\frac{\log A^3 B^3}{C}$

B) $\log \left(\frac{AB}{C} \right)^3$

C) $\log \frac{(AB)^3}{C}$

D) $\log \frac{AB^3}{C}$

10. What is the value of $\log_2(4x)$ if $\log_2 x = 3$?

- A) 5
B) 6
C) 7
D) 12

$$\begin{array}{l|l} 2^3 = x & \log_2 4(8) \\ x = 8 & \\ \hline & 2^x = 32 \\ & x = 5 \end{array}$$

10. A

11. If $\log_2 9 = x$, then $\log_2 \sqrt[3]{9^3}$ is equivalent to which expression?

- A) $x^{\frac{3}{5}}$
B) $x^{\frac{5}{3}}$
C) $\frac{3x}{5}$
D) $\frac{5x}{3}$

$$\begin{array}{l|l} 2^x = 9 & \log_2(9)^{3/5} \\ \hline & \frac{3}{5}x \end{array}$$

11. C

12. What is the x-intercept of $y = \log_2(x+7)$?

- A) 1
B) -6
C) 8
D) -7

$$0 = \log_2(x+7)$$

$$2^0 = x+7$$

$$1 = x+7$$

$$x = -6$$

12. B

13. What is the inverse of $y = 4^x$?

- A) $y = \log_x 4$
B) $y = \log_4 x$
C) $x = \log_4 y$
D) $x = \log_y 4$

$$x = 4^y$$

$$\log_4 x = y$$

13. B

14. \$3500 is invested in an account that pays 5.5% interest compounded quarterly. What is the balance after 8 years?

- A) \$5418.21
B) \$8338.47
C) \$3904.05
D) \$10 836.42

$$\begin{aligned} y &= 3500 \left(1 + \frac{0.055}{4}\right)^{32} \\ y &= 3500 (1.01375)^{32} \\ y &= 5418.21 \end{aligned}$$

14. A

15. Solve for x: $4^{x+1} = 5(3^{2x})$

- A) $\frac{\log 5 - \log 4}{1 - 2 \log 3}$
B) $\frac{\log 5 - \log 4}{\log 4 - 2 \log 3}$
C) $\frac{-\log 4}{\log 4 - 2 \log 15}$
D) $\frac{-\log 4}{1 - 2 \log 15}$

15. B

Section B: Constructed Response. Be sure to show all workings in order to receive full marks.
(17 marks)

16. Solve for x : $\log_8(6x+2) + \log_8(x-3) = 2$ (5 marks)

$$\begin{aligned} \log_8(6x+2)(x-3) &= 2 \\ 8^2 &= 6x^2 - 16x - 6 \\ 0 &= 6x^2 - 16x - 70 \\ 0 &= 2(3x^2 - 8x - 35) \\ 0 &= 2(3x+7)(x-5) \\ x &= -7/3 \quad \text{or} \quad x = 5 \\ \text{reject} \end{aligned}$$

17. The half life of plutonium-238 is 88 years. Suppose that a sample of plutonium has a mass of 65 grams. Write an exponential function and determine the time needed for the sample to

decay to a mass of 20 grams. $\left[A(t) = A_0 \left(\frac{1}{2} \right)^{t/h} \right]$

$$\begin{aligned} 20 &= 65 \left(\frac{1}{2} \right)^{x/88} && (4 \text{ marks}) \\ \frac{20}{65} &= \left(\frac{1}{2} \right)^{x/88} \\ \left(\frac{4}{13} \right) &= \left(\frac{1}{2} \right)^{x/88} \\ \log \left(\frac{4}{13} \right) &= \frac{x}{88} \log \left(\frac{1}{2} \right) \\ \frac{x}{88} &= \frac{\log \left(\frac{4}{13} \right)}{\log \left(\frac{1}{2} \right)} \\ x &= \frac{88 \log \left(\frac{4}{13} \right)}{\log \left(\frac{1}{2} \right)} \\ x &= 149.64 \end{aligned}$$

18. One used car costs \$6000 and depreciates in value at 5% every 3 years, while another used car costs \$9000 and depreciates in value at a rate of 8% every 2 years. If both cars were purchased at the same time, when will the value of both be the same?

$$6000(0.95)^{x/3} = 9000(0.92)^{x/2}$$

(5 marks)

$$\log 6000 + \frac{x}{3} \log 0.95 = \log 9000 + \frac{x}{2} \log 0.92$$

$$\frac{x}{3} \log 0.95 - \frac{x}{2} \log 0.92 = \log 9000 - \log 6000$$

$$x \left[\frac{1}{3} \log 0.95 - \frac{1}{2} \log 0.92 \right] = \log 9000 - \log 6000$$

$$x = \frac{\log 9000 - \log 6000}{\frac{1}{3} \log 0.95 - \frac{1}{2} \log 0.92} = \frac{0.176091}{0.0106806} = 16.49$$

19. The intensity level β in decibels of a sound is defined by $\beta = 10(\log I + 12)$ where I is the intensity of the sound in watts per square metre. A fire truck siren has a decibel level of 118 dB. City traffic has a decibel level of 85dB. How many times as loud as city traffic is the fire truck siren?

$$118 = 10(\log I + 12)$$

$$11.8 = \log I + 12$$

$$-0.2 = \log I$$

$$10^{-0.2} = I$$

$$85 = 10(\log I + 12)$$

$$8.5 = \log I + 12$$

$$-3.5 = \log I$$

$$10^{-3.5} = I$$

(3 marks)

$$\frac{10^{-3.5}}{10^{-0.2}} = 10^{-3}$$