Chapter 6 Review Sheet

1a) $\frac{\sin x}{\cos x}$ NPV : $x \neq \frac{\pi}{2} + \pi n, n \in I$

b) $\cos x \neq 1$

$x \neq 0 + 2\pi n, n \in I$

c) $\sin x \neq -1$

$x \neq \frac{3\pi}{2} + 2\pi n, n \in I$

#2. $\sin\theta \neq 0$

$\theta \neq 0 + \pi n, n \in I$

$\cos\theta \neq 1$

$\theta \neq 0 + 2\pi n, n \in I$

Restriction

$\theta \neq \pi n, n \in I$

#3. $\frac{\cot x}{\csc x} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

$= \frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{1}$

$= \cos x$

b) $\frac{1}{\cot x \sec x} = \frac{1}{\frac{\cancel{\cos x}}{\sin x} \cdot \frac{1}{\cancel{\cos x}}}$

$= \sin x$

c) $\frac{1 - \tan x}{\cot x - 1}$

$= \frac{1 - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} - 1}$

$= \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x - \sin x}{\sin x}}$

$= \frac{\cancel{\cos x - \sin x}}{\cos x} \cdot \frac{\sin x}{\cancel{\cos x - \sin x}}$

$\frac{\sin x}{\cos x}$

$= \tan x$

#5. $2(\csc^2 x - \cot^2 x)$

$= 2\left(\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}\right)$

$= 2\left(\frac{1-\cos^2 x}{\sin^2 x}\right)$

$= \frac{2\,(\cancel{\sin^2 x})}{\cancel{\sin^2 x}}$

$= 2$

b) $\cot^2 x(\sec^2 x - 1)$

$= \cot^2 x\,(\tan^2 x)$

$= \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x}$

$= 1$

c) $\frac{\sin^2 x}{\cos^2 x} + \sin x \csc x$

$= \frac{\sin^2 x}{\cos^2 x} + \sin x \frac{1}{\sin x}$

$= \tan^2 x + 1$

$= \sec^2 x$

d) $\frac{\cos x}{\cancel{\sin x}\,\frac{\cos x}{\cancel{\sin x}}} = 1$

e) $\tan x \cos^2 x$

$= \frac{\sin x}{\cos x} \cdot \cos^2 x$

$= \sin x \cos x$

F) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x}$

$= \cos^2 x + \sin^2 x$

$= 1$

6. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}$

$= \frac{\frac{1}{\cos x}}{\sin x} - \frac{\sin x}{\cos x}$

$= \frac{1}{\cos x \sin x} - \frac{\sin x}{\cos x}$

$= \frac{1 - \sin^2 x}{\sin x \cos x}$

$= \frac{\cos^2 x}{\sin x \cos x}$

$= \cot x$

b) $\cos x + \tan x \sin x$

$= \cos x + \frac{\sin x}{\cos x} \sin x$

$= \frac{\cos^2 x + \sin^2 x}{\cos x}$

$= \frac{1}{\cos x}$

$= \sec x$

c) $\sin x + \cos x \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin x}$

$= \frac{1}{\sin x} = \csc x$

#7. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

$\sin^4 \frac{\pi}{4} - \cos^4 \frac{\pi}{4} \stackrel{?}{=} 2\sin^2\left(\frac{\pi}{4}\right) - 1$

$\left(\frac{\sqrt{2}}{2}\right)^4 - \left(\frac{\sqrt{2}}{2}\right)^4 \stackrel{?}{=} 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1$

$\frac{4}{16} - \frac{4}{16} \stackrel{?}{=} 2\left(\frac{2}{4}\right) - 1$

$0 = 0$

#8 $\sin(28+35)$

$\sin 63^\circ$

b) $\cos\left(\frac{\pi}{12} - \frac{\pi}{4}\right)$

$\cos\left(\frac{-\pi}{6}\right)$

#9. $\cos(30°) = \frac{\sqrt{3}}{2}$

b) $\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$\sin\frac{\pi}{2} = 1$

#10. $2\sin\frac{\pi}{6}\cos\frac{\pi}{6}$

$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{2}$

b) $\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{4}$

$= -\frac{1}{2}$

#11 $\sin(\overset{A}{90°} + \overset{B}{A})$

$= \sin 90 \cos A + \cos 90 \sin B$

$= \cos A$

b) $\cos(2\pi + A)$

$= \cos 2\pi \cos A - \sin 2\pi \sin A$

$= \cos A$

#12 $\frac{\sin 2\theta}{2\sin\theta} = \frac{2\cancel{\sin\theta}\cos\theta}{2\cancel{\sin\theta}}$

$= \cos\theta$

b) $\cos 3x \cos x - \sin 3x \sin x$

$\cos 4x$

c) $\frac{\cos 2\theta - 1}{2\sin\theta} = \frac{1 - 2\sin^2\theta - 1}{2\sin\theta}$

$= \frac{-2\sin^2\theta}{2\sin\theta}$

$= -\sin\theta$

d) $\frac{\sin^3 x}{\cos 2x - \cos^2 x} = \frac{\sin^3 x}{\cos^2 x - \sin^2 x - \cos^2 x}$

$= \sin x$

#13. $\cos \frac{2\pi}{3}$

$$\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = \cos\frac{\pi}{2}\cos\frac{\pi}{6} - \sin\frac{\pi}{2}\sin\frac{\pi}{6}$$

$$= 0 - 1\left[\frac{1}{2}\right)$$

$$= -1/2$$

b) $\tan 15^\circ = \tan(45-30)$

$$\tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$$

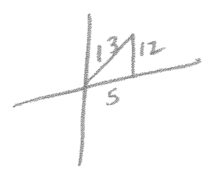
$$= \frac{3-\sqrt{3}}{3} \cdot \frac{3}{3+\sqrt{3}}$$

$$= \frac{3-\sqrt{3}}{3+\sqrt{3}}$$

☆#15 $\sin A = \frac{3}{5}$ $\left(\frac{y}{r}\right)$

$\cos A = \frac{4}{5}$

$\cos B = \frac{5}{13}$ $\left(\frac{x}{r}\right)$



$\sin B = \frac{12}{13}$

a) $\cos(A-B) = \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$

$= -16/65$

#16. $\cos A = \frac{12}{13}$ $\left(\frac{x}{r}\right)$

$y = 5$ fourth quadrant $\therefore$ $\sin A = \frac{-5}{13}$

$\sin 2A = 2\sin A \cos A$

$= 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \frac{-120}{169}$

#17. $\dfrac{\tan x(1-\sin^2 x)}{\cos^2 x} = \tan x$

b) $\dfrac{(\sin x \cancel{+3})(\sin x - 2)}{5(\sin x \cancel{+3})} = \dfrac{\sin x - 2}{5}$

c) $\dfrac{(\cos x \cancel{-2})(\cos x+2)}{7(\cos x \cancel{-2})} = \dfrac{\cos x + 2}{7}$

d) $\dfrac{\tan x(\sin^2 x - 1)}{\tan x(\sin x + 1)} = \dfrac{(\sin x - 1)\cancel{(\sin x + 1)}}{\cancel{(\sin x + 1)}} = \sin x - 1$

#18. $\csc^2 x(1-\cos^2 x)$

$\csc^2 x \sin^2 x$

$\cancel{\csc^2 x} \dfrac{1}{\cancel{\csc^2 x}}$

1

b) $(\tan x - 1)(\tan x - 1)$

$\tan^2 x - 2\tan x + 1$

$\sec^2 x - 2\tan x$

$\sec^2 x - 2\tan x$

c) $\dfrac{\sin^2 x + \cos^2 x}{\sec x}$

$\dfrac{[1-\cos^2 x] + \cos^2 x}{\sec x}$

$\dfrac{1}{\sec x}$

$\cos x$

$\cos x$

d) $\dfrac{1+\tan x}{1+\cot x}$

$= \dfrac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}}$

$= \dfrac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}}$

$= \dfrac{\cancel{\cos x + \sin x}}{\cos x} \cdot \dfrac{\sin x}{\cancel{\sin x + \cos x}}$

$= \tan x$

$\tan x$

e) $\dfrac{\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}}{}$ | $\cot x$

$= \dfrac{\sec x \cos x - \sin^2 x}{\sin x \cos x}$ | $\dfrac{\cos x}{\sin x}$

$= \dfrac{1-\sin^2 x}{\sin x \cos x}$

$= \dfrac{\cos^2 x}{\sin x \cos x}$

$= \dfrac{\cos x}{\sin x}$

f) $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

$= \dfrac{\frac{1}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \sin x}$ | $\dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x}$

$= \dfrac{\frac{1+\cos x}{\sin x}}{\frac{\sin x + \sin x \cos x}{\sin x}}$ | $\dfrac{1}{\sin x}$

$= \dfrac{1+\cos x}{\sin x + \sin x \cos x}$

$= \dfrac{1+\cancel{\cos x}}{\sin x(1+\cancel{\cos x})}$

$= \dfrac{1}{\sin x}$

g) $\dfrac{\sin x + \tan x}{\cos x + 1}$ | $\tan x$

$= \dfrac{\sin x + \frac{\sin x}{\cos x}}{\cos x + 1}$ | $\dfrac{\sin x}{\cos x}$

$= \dfrac{\frac{\sin x \cos x + \sin x}{\cos x}}{\cos x + 1}$

$= \dfrac{\frac{\sin x(\cos x + 1)}{\cos x}}{\cos x + 1}$

$= \dfrac{\sin x}{\cos x}$

h) $\dfrac{\cos x + 1}{\sin x + \tan x} = \cot x$

$= \dfrac{\cos x + 1}{\sin x + \frac{\sin x}{\cos x}}$ | $\dfrac{\cos x}{\sin x}$

$= \dfrac{\cos x + 1}{\frac{\sin x \cos x + \sin x}{\cos x}}$

$= (\cancel{\cos x + 1}) \cdot \dfrac{\cos x}{\sin x(1+\cancel{\cos x})}$

$= \dfrac{\cos x}{\sin x}$

i) $\dfrac{\cos x}{1-\sin x} = \dfrac{1+\sin x}{\cos x}$

$\dfrac{\cos x}{1-\sin x}\left[\dfrac{1+\sin x}{1+\sin x}\right]$

$= \dfrac{\cos x + \sin x \cos x}{1-\sin^2 x}$

$= \dfrac{\cos x(1+\sin x)}{\cos^2 x}$

$= \dfrac{1+\sin x}{\cos x}$ | $\dfrac{1+\sin x}{\cos x}$

j) $\dfrac{1+\cos x}{\sin x} = \dfrac{\sin x}{1-\cos x}$

$= \dfrac{\sin x}{1-\cos x}\cdot\left[\dfrac{1+\cos x}{1+\cos x}\right]$

$= \dfrac{\sin x(1+\cos x)}{1-\cos^2 x}$

$= \dfrac{\sin x(1+\cos x)}{\sin^2 x}$

$= \dfrac{1+\cos x}{\sin x}$

k) $\dfrac{\cos x}{\sec x - 1} + \dfrac{\cos x}{\sec x + 1} = 2\cot^2 x$

$\dfrac{\cos x}{\sec x-1}\left[\dfrac{\sec x+1}{\sec x+1}\right] + \dfrac{\cos x}{\sec x+1}\left[\dfrac{\sec x-1}{\sec x-1}\right]$

$\dfrac{\cos x\sec x + \cos x + \cos x \sec x - \cos x}{\sec^2 x - 1}$

$\dfrac{2}{\tan^2 x}$

$= 2\cot^2 x$

l) No solution

m) $\dfrac{1+\cos 2x}{\sin 2x} = \cot x$

$= \dfrac{1+2\cos^2 x - 1}{2\sin x\cos x}$ | $\dfrac{\cos x}{\sin x}$

$= \dfrac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x \cancel{\cos x}}$

$= \dfrac{\cos x}{\sin x}$

n) $1+\sin 2x = (\sin x + \cos x)^2$

$1 + 2\sin x\cos x$ | $(\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$1 + 2\sin x\cos x$

o) $\sec^2 x = \dfrac{2}{1+\cos 2x}$

$= \dfrac{2}{1+[\cos^2 x - \sin^2 x]}$

$= \dfrac{2}{1+\cos^2 x - \sin^2 x}$

$= \dfrac{2}{2\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$

$= \sec^2 x$

p) No solution

#19 $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

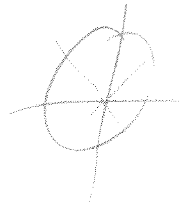
$\cos x = 0$ $\sin x = 1/2$

$x = \pi, \frac{3\pi}{2}$ $x = \pi/6, \frac{5\pi}{6}$

b) $2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos = \pm\frac{\sqrt{2}}{2}$



$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

c) $\cos^2 x - 2 - \cos x = 0$

$\cos^2 x - \cos x - 2 = 0$

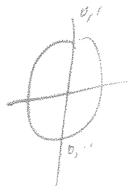
$(\cos x - 2)(\cos x + 1) = 0$

$\cos x = -1$

$x = \pi$

c) $\cot^2 x = 0$

$x = \pi, \frac{3\pi}{2}$



#20 a) can't do

b) $2\cos^2 x - 5\sin x - 5 = 0$

$2[1 - \sin^2 x] - 5\sin x - 5 = 0$

$2 - 2\sin^2 x - 5\sin x - 5 = 0$

$-3 - 2\sin^2 x - 5\sin x = 0$

$2\sin^2 x + 5\sin x + 3 = 0$

$(2\sin x + 3)(\sin x + 1) = 0$

$\cancel{\sin x = \frac{-3}{2}}$ or $\sin x = -1$

$x = \frac{3\pi}{2}$

#21 a) $\cos 2x - 5\cos x = 2$

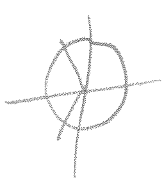
$2\cos^2 x - 1 - 5\cos x - 2 = 0$

$2\cos^2 x - 5\cos x - 3 = 0$

$(2\cos x + 1)(\cos x - 3) = 0$

$\cos x = -\frac{1}{2}$ $\quad \cancel{\cos x = 3}$

$x = \frac{2\pi}{3}$ & $\frac{4\pi}{3}$



b) $\cot^2 x + 2 = 0$

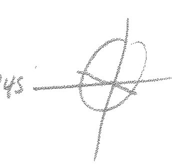
$\cot^2 x = -2$

$\tan^2 x = -\frac{1}{2}$

$\tan x = \pm \frac{\sqrt{2}}{2}$

Not an exact value!!

$x = 35°, 215°$ | $x = 325°, 145°$



0.6109, 3.7525 $\quad$ 5.6723, 2.5307

c) $1 + \cos x = 2[1 - \cos^2 x]$

$1 + \cos x = 2 - 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$ $\quad$ No solution

$\cancel{(2\cos x \quad)(? \quad) = 0}$

22. $\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\cancel{x} = 45°, 135°, \cancel{225°}, \cancel{315°}, -45°, -135°$