Applications of Logarithms – Chapter 8

Sound is measured using a logarithmic scale. The decibel level, B, is defined as:

$B = 10 \log \frac{I}{I_0}$ ← intensity of sound

I_0 ← 10^{-12} (reference sound)

Example 1: If one siren emits a noise at a decibel level of 100, how much 'noise' is generated by 3 sirens at the same time? (find decibel level of 3 sirens)

$100 = 10 \log \left(\frac{I}{10^{-12}}\right)$

$10 = \log \left(\frac{I}{10^{-12}}\right)$

$10^{10} = \frac{I}{10^{-12}}$

$(10^{10})(10^{-12}) = I$

$I = 10^{-2}$

$B = 10 \log \frac{3 \cdot 10^{-2}}{10^{-12}}$

$B = 10 \log [3(10^{10})]$

$= 104.771$

Example 2: One noise measured 95 decibels and a 2nd noise measured 1/10 the intensity of the 1st noise. What would be the decibel level for the second noise?

1st

$$95 = 10 \log \frac{I}{10^{-12}}$$

$$9.5 = \log \frac{I}{10^{-12}}$$

$$10^{9.5} = \frac{I}{10^{-12}}$$

$$10^{9.5}\, 10^{-12} = I$$

$$10^{-2.5} = I$$

2nd

$$B = 10 \log \frac{(1/10)(10^{-2.5})}{10^{-12}}$$

$$B = 10 \log \frac{(10^{-1})(10^{-2.5})}{10^{-12}}$$

$$B = 10(8.5)$$

$$B = 85$$

$\log_{10} 10^{8.5} = 8.5$

$10^{y} = 10^{8.5}$

Example 3:
1st Noise – 72 decibel 2nd noise – 92 decibel
How much more intense is the second noise as compared to the first noise?

$72 = 10 \log \frac{I}{10^{-12}}$

$7.2 = \log \frac{I}{10^{-12}}$

$10^{7.2} = \frac{I}{10^{-12}}$

$(10^{7.2})10^{-12} = I$

$10^{-4.8} = I$

$92 = 10 \log \frac{I}{10^{-12}}$

$\vdots$

$I = 10^{-2.8}$

$\frac{10^{-2.8}}{10^{-4.8}} = 10^{2}$

$= 100$

The Richter scale is a logarithmic scale used to measure the strength of an earthquake. The formula is:

$$M = \log\left(\frac{A}{A_0}\right)$$ ← amplitude / intensity
← reference 1

Example 1: In 1989 there was an earthquake in San Francisco. The magnitude of the quake was 6.9 on the Richter Scale. In 2010 there was a catastrophic earthquake in Haiti. It's magnitude was 7.0. how much more intense was the Haiti earthquake that the San Francisco one?

SF

$6.9 = \log\left(\frac{A}{1}\right)$

$10^{6.9} = A$

H

$7 = \log A$

$10^7 = A$

$\frac{10^7}{10^{6.9}} = 1.26$

Example 2: In 1960 Chili had an earthquake with a magnitude of 9.5 on the Richter Scale. How much stronger was this earthquake as compared to Haiti's earthquake in 2010.

Example 3: What was the magnitude of the earthquake that hit the Ottawa area in 2013 if it had 10^{-2} (1/100) the intensity of the earthquake that hit the Yukon/Alaskan border in 1970?

$M = 7.2$

Yukon

$7.2 = \log A$

$10^{7.2} = A$

Ottawa

$M = \log\left(10^{-2} \cdot 10^{7.2}\right)$

$M = \log 10^{5.2}$

$M = 5.2$

Example 2
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