$y = af[b(x - h)] + k$

Transformations and Operations
LESSON TWO - *Combined Transformations*
Lesson Notes

Example 1 Answer the following questions:

Combining Stretches and Reflections

a) Identify each parameter in the general transformation equation: $y = af[b(x - h)] + k$.

$a \Rightarrow$ vertical stretch

$b \Rightarrow \frac{1}{b}$ horizontal stretch

$h \Rightarrow$ horizontal translation

$k \Rightarrow$ vertical translation

b) Describe the transformations in each equation: & write the mapping rule.

i) $y = \frac{1}{3}f(5x)$

vertical stretch $\frac{1}{3}$

horizontal stretch $\frac{1}{5}$

$(x, y) \rightarrow (\frac{1}{5}x, \frac{1}{3}y)$

ii) $y = 2f(\frac{1}{4}x)$

vertical stretch 2

Horizontal stretch 4

$(x, y) \rightarrow (4x, 2y)$

iii) $y = -\frac{1}{2}f(\frac{1}{3}x)$

Vertical stretch 1/2

horizontal stretch 3

refection over the x-axis

$(x, y) \rightarrow (3x, -\frac{1}{2}y)$

iv) $y = -3f(-2x)$

vertical stretch 3

Horizontal stretch $\frac{1}{2}$

reflection over the x & y

$(x, y) \rightarrow (-\frac{1}{2}x, -3y)$

Transformations and Operations

LESSON TWO - *Combined Transformations*

Lesson Notes

y = af[b(x - h)] + k

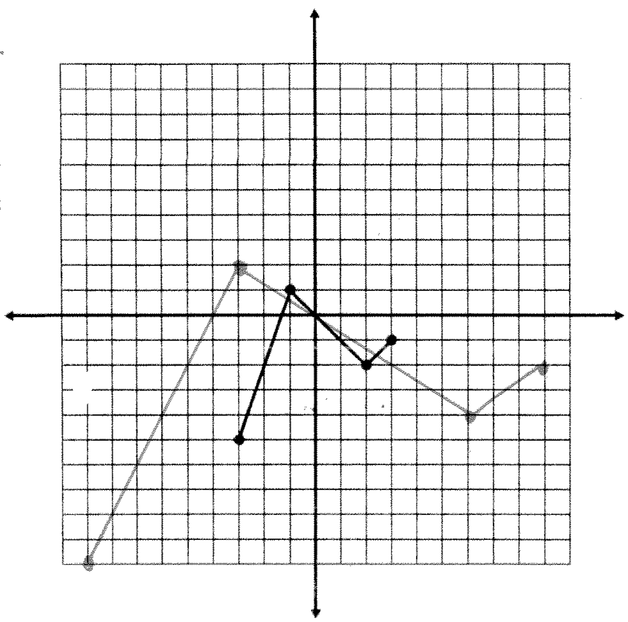
Example 2 Draw the transformation of each graph. & state mapping rule.

Combining Stretches and Reflections

a) $y = 2f(\frac{1}{3}x)$ $(x, y) \rightarrow (3x, 2y)$

x	y
-3	-5
-1	1
2	-2
3	-1

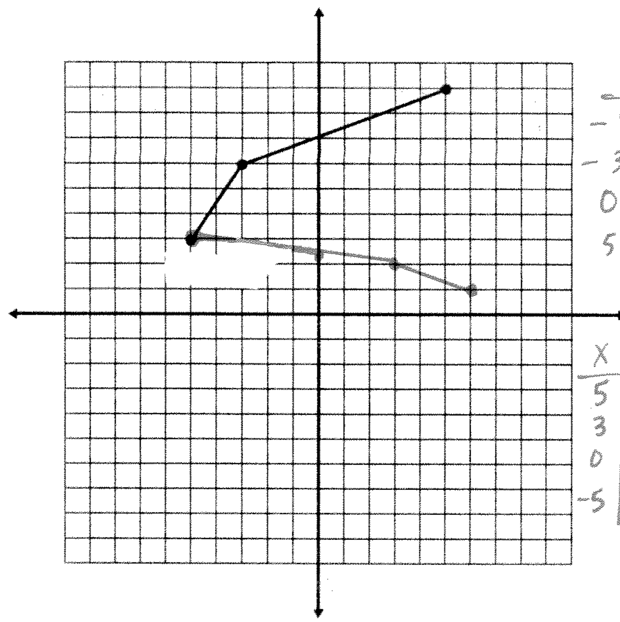
3x	2y
-9	-10
-3	2
6	-4
9	-2



b) $y = \frac{1}{3}f(-x)$ $(x, y) \rightarrow (-x, \frac{1}{3}y)$

x	y
-5	3
-3	6
0	7
5	9

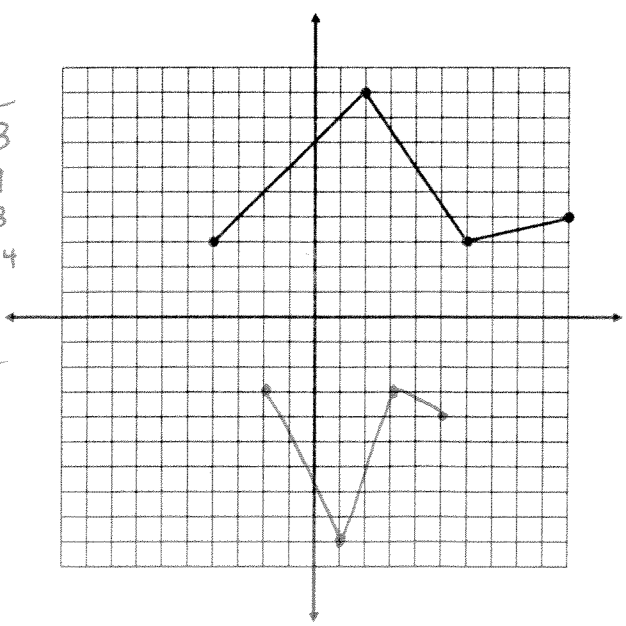
x	y
5	1
3	2
0	7/3
-5	3



c) $y = -f(2x)$ $(x, y) \rightarrow (\frac{1}{2}x, -y)$

x	y
-4	3
2	9
6	3
10	4

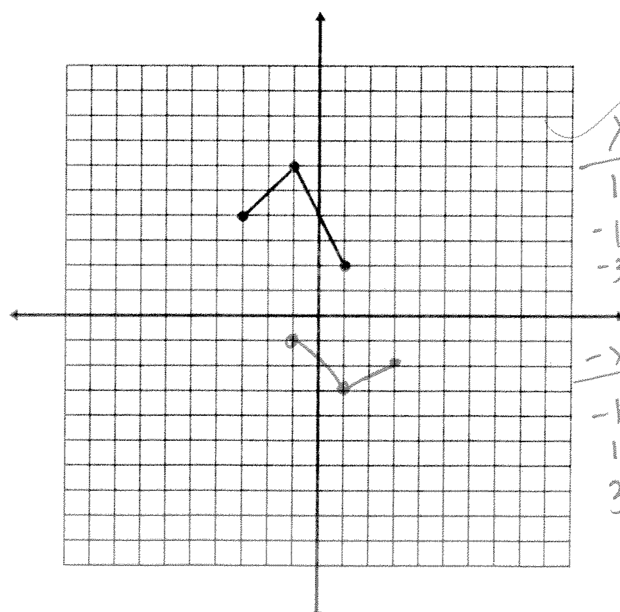
$\frac{1}{2}x$	-y
-2	-3
1	-9
3	-3
5	-4



d) $y = -\frac{1}{2}f(-x)$ $(x, y) \rightarrow (-x, -\frac{1}{2}y)$

x	y
1	2
-1	6
-3	4

-x	$-\frac{1}{2}y$
-1	-1
1	-3
3	-2



$y = af[b(x - h)] + k$

Transformations and Operations
LESSON TWO - *Combined Transformations*
Lesson Notes

Example 3 Answer the following questions:

Combining Translations

a) ~~Find the horizontal translation of y = f(x + 3) using three different methods.~~

~~Opposite Method:~~ ~~Zero Method:~~ ~~Double Sign Method:~~

b) Describe the transformations in each equation. State mapping rule.

i) $y = f(x - 1) + 3$

$(x, y) \to (x+1, y+3)$

VT 3

HT 1

ii) $y = f(x + 2) - 4$

$(x, y) \to (x-2, y-4)$

VT -4

HT -2

iii) $y = f(x - 2) - 3$

$(x, y) \to (x+2, y-3)$

VT -3

HT 2

iv) $y = f(x + 7) + 5$

$(x, y) \to (x-7, y+5)$

VT 5

HT -7

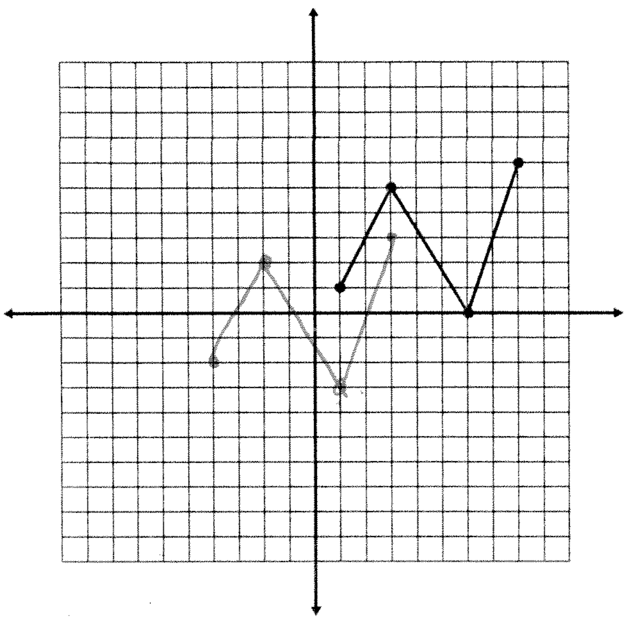
Transformations and Operations
LESSON TWO - *Combined Transformations*
Lesson Notes

$y = af[b(x - h)] + k$

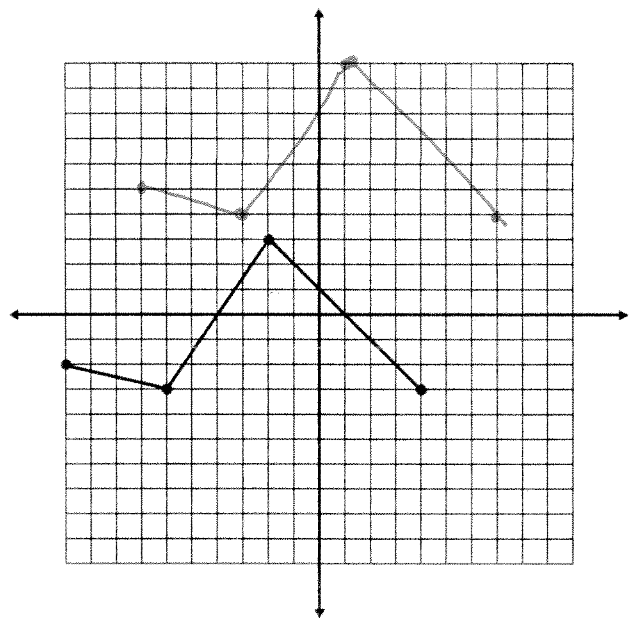
Example 4 Draw the transformation of each graph. state mapping rule

Combining Translations

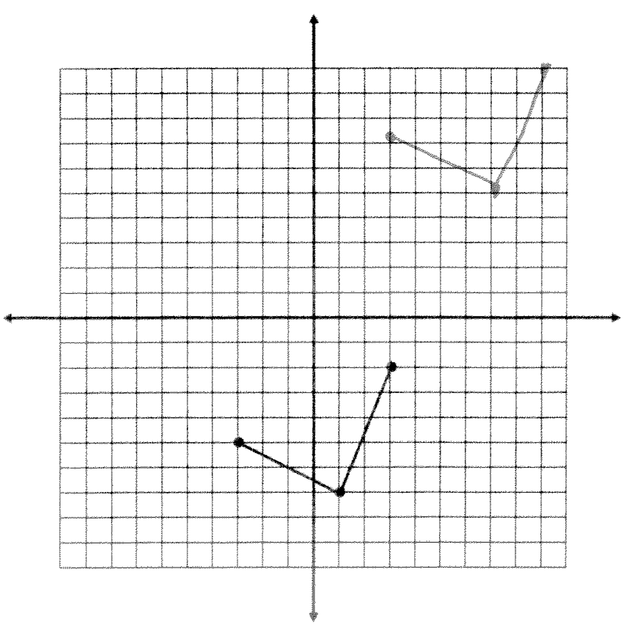
a) $y = f(x + 5) - 3$ $(x, y) \to (x-5, y-3)$



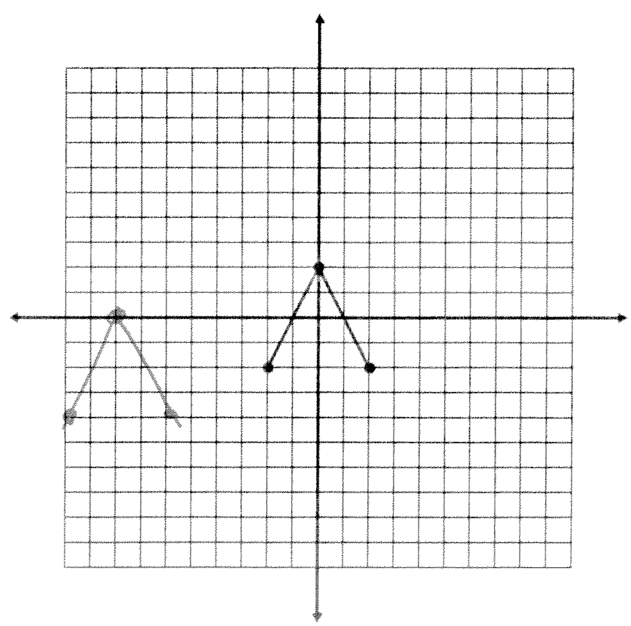
b) $y = f(x - 3) + 7$ $(x, y) \to (x+3, y+7)$



c) $y - 12 = f(x - 6)$ $(x, y) \to (x+6, y+12)$



d) $y + 2 = f(x + 8)$ $(x, y) \to (x-8, y-2)$



$y = af[b(x - h)] + k$

Transformations and Operations
LESSON TWO - *Combined Transformations*
Lesson Notes

Example 5 Answer the following questions:

Combining Stretches, Reflections, and Translations

a) When applying transformations to a graph, should they be applied in a specific order?

stretches, reflections, Translations

RST

b) Describe the transformations in each equation. & state mapping rule

i) $y = 2f(x + 3) + 1$

$(x, y) \to (x - 3, 2y + 1)$

VT 1

VS 2

HT -3

ii) $y = -f(\frac{1}{3}x) - 4$

$(x, y) \to (3x, -y - 4)$

VT -4

HS 3

iii) $y = \frac{1}{2}f[-(x + 2)] - 3$

$(x, y \to (-x - 2, \frac{1}{2}y - 3)$

VS $\frac{1}{2}$

VT -3

HT -2

Reflection over the y-axis

iv) $y = -3f[-4(x - 1)] + 2$

$(x, y) \to (-\frac{1}{4}x + 1, -3y + 2)$

VS 3

VT 2

HS 1/4

HT 1

reflection over x & y

Transformations and Operations

LESSON TWO - *Combined Transformations*

Lesson Notes

$$y = af[b(x - h)] + k$$

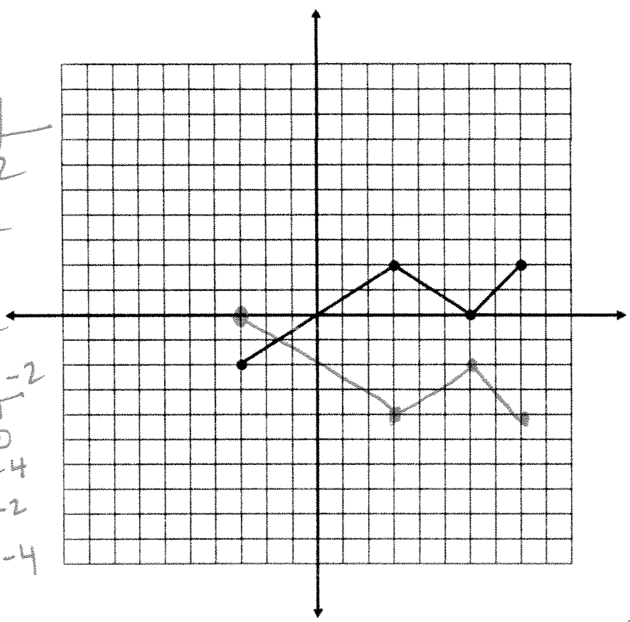
Example 6 Draw the transformation of each graph. & state mapping rule

Combining Stretches, Reflections, and Translations

a) $y = -f(x) - 2$ $(x, y) \rightarrow (x, -y - 2)$

x	y
-3	-2
3	2
6	0
8	2

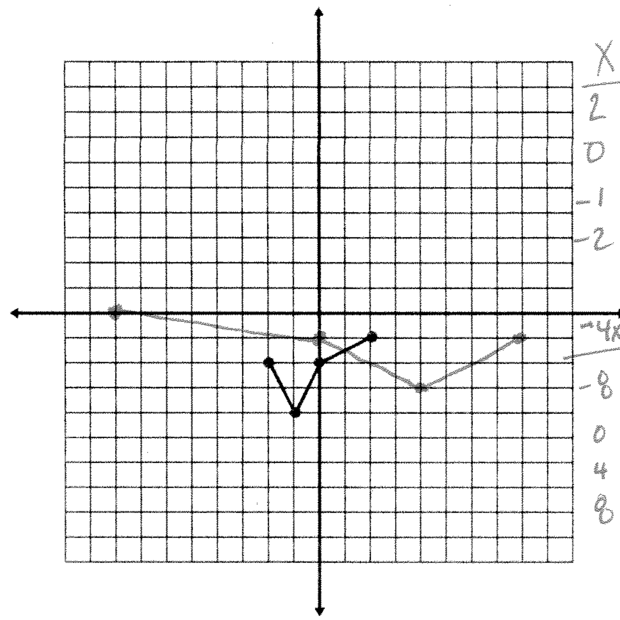
x	-y-2
-3	0
3	-4
6	-2
8	-4



b) $y = f(-\frac{1}{4}x) + 1$ $(x, y) \rightarrow (-4x, y + 1)$

x	y
2	-1
0	-2
-1	-4
-2	-2

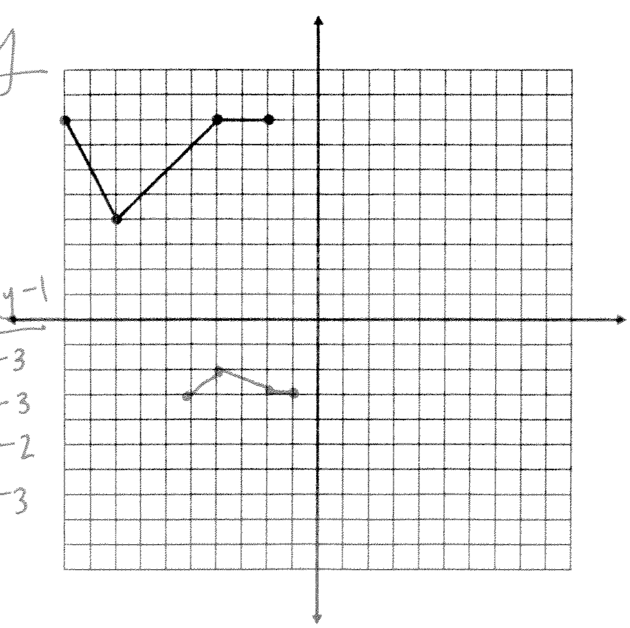
-4x	y+1
-8	0
0	-1
4	-3
8	-1



c) $y = -\frac{1}{4}f(2x) - 1$ $(x, y) \rightarrow (\frac{1}{2}x, -\frac{1}{4}y - 1)$

x	y
-2	8
-4	8
-8	4
-10	8

$\frac{1}{2}x$	$-\frac{1}{4}y-1$
-1	-3
-2	-3
-4	-2
-5	-3



d) $2y - 8 = 6f(x - 2)$

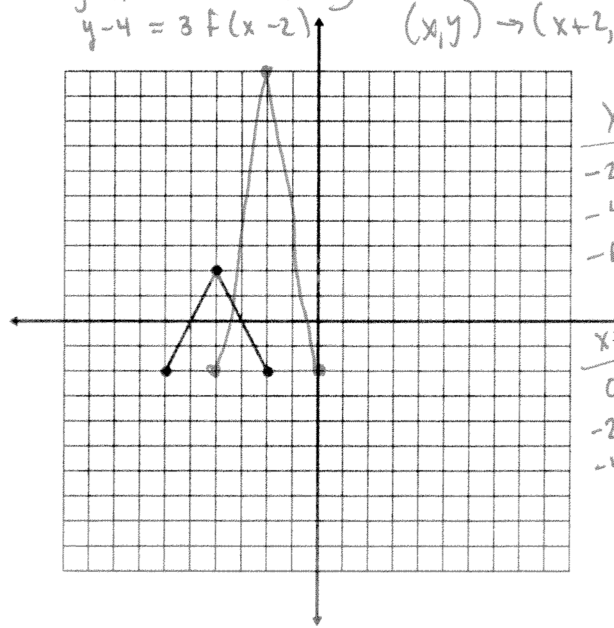
$2(y-4) = 6f(x-2)$

$y - 4 = 3f(x-2)$ $\rightarrow$ $y = 3f(x-2) + 4$

$(x, y) \rightarrow (x + 2, 3y + 4)$

x	y
-2	-2
-4	2
-6	-2

x+2	3y+4
0	-2
-2	10
-4	-2



$y = af[b(x - h)] + k$

Transformations and Operations
LESSON TWO - *Combined Transformations*
Lesson Notes

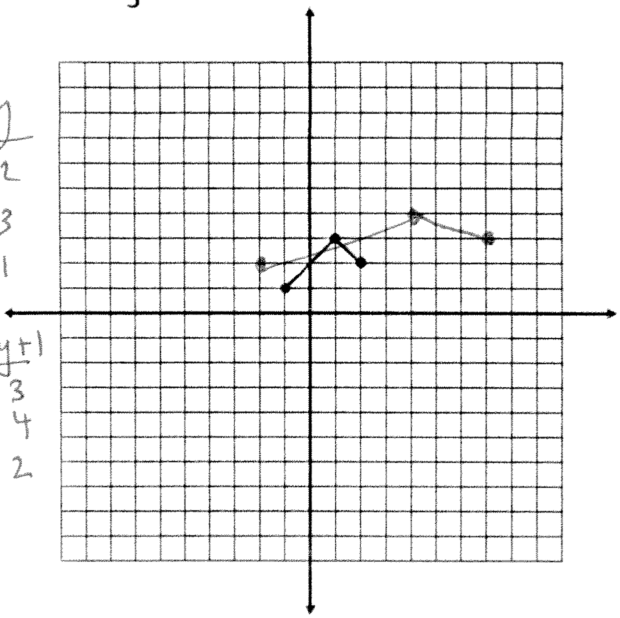
Example 7 Draw the transformation of each graph. + state mapping rule

Combining Stretches, Reflections, and Translations
(watch for b-factoring)

a) $y = f[\frac{1}{3}(x - 1)] + 1$ $(x, y) \to (3x+1, y+1)$

x	y
2	2
1	3
-1	1

3x+1	y+1
7	3
4	4
-2	2

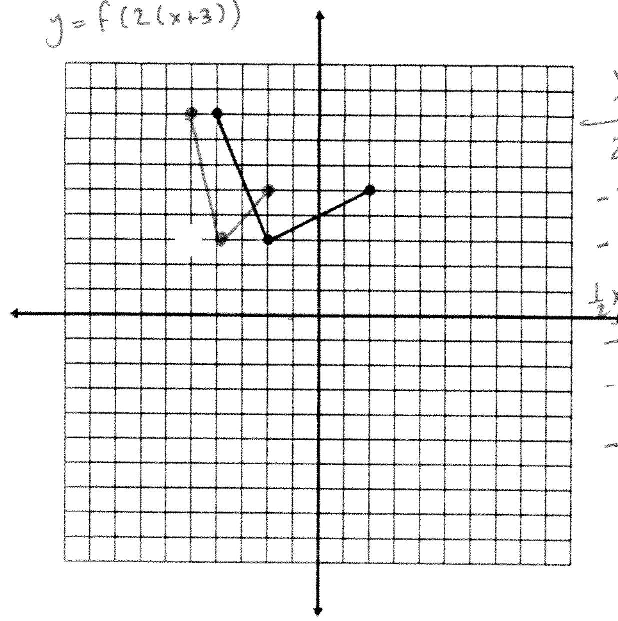


b) $y = f(2x + 6)$ $(x, y) \to (\frac{1}{2}x - 3, y)$

$y = f(2(x+3))$

x	y
2	5
-2	3
-4	8

$\frac{1}{2}x-3$	y
-2	5
-4	3
-5	8

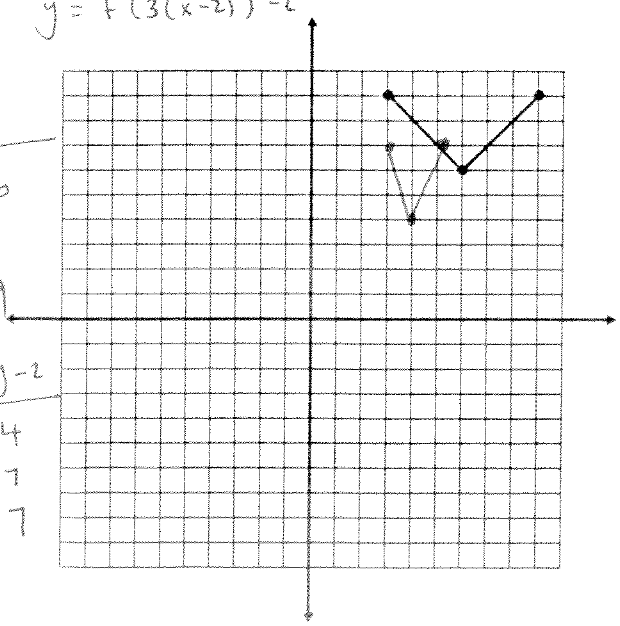


c) $y = f(3x - 6) - 2$ $(x, y) \to (\frac{1}{3}x + 2, y - 2)$

$y = f(3(x-2)) - 2$

x	y
6	6
9	9
3	9

$\frac{1}{3}x+2$	y-2
4	4
5	7
3	7

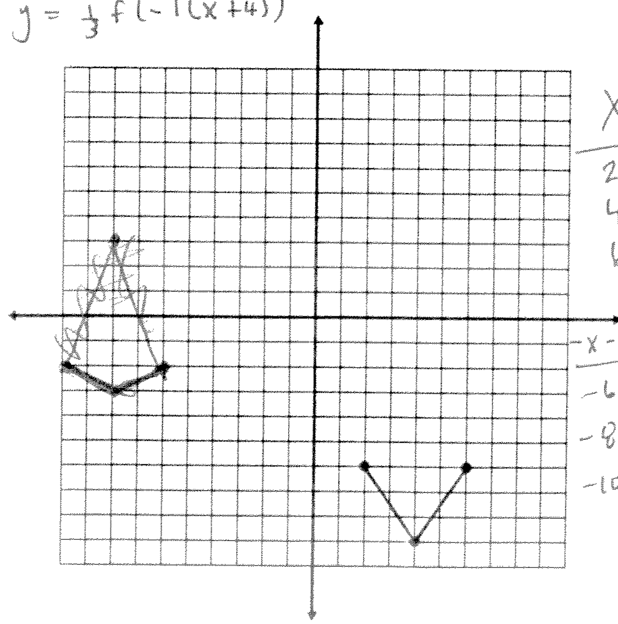


d) $y = \frac{1}{3}f(-x - 4)$ $(x, y) \to (-x-4, \frac{1}{3}y)$

$y = \frac{1}{3}f(-1(x+4))$

x	y
2	-6
4	-9
6	-6

-x-4	$\frac{1}{3}y$
-6	-2
-8	-3
-10	-2



Transformations and Operations

LESSON TWO - *Combined Transformations*

Lesson Notes

$$y = af[b(x - h)] + k$$

Example 8 Answer the following questions: **Mappings**

The mapping for combined transformations is:

$$\boxed{(x, y) \rightarrow \left(\frac{x_i}{b} + h,\ ay_i + k\right)}$$

a) If the point (2, 0) exists on the graph of y = f(x), find the coordinates of the new point after the transformation y = f(-2x + 4).

$y = f(-2(x-2))$

$(x, y) \rightarrow \left(-\frac{1}{2}x + 2, y\right)$

$\rightarrow \left(-\frac{1}{2}(2) + 2, 0\right)$

$\rightarrow (1, 0)$

$(1, 0)$

b) If the point (5, 4) exists on the graph of y = f(x), find the coordinates of the new point after the transformation $y = \frac{1}{2}f(5x - 10) + 4$.

$5(x-2)$

$(x, y) \rightarrow \left(\frac{1}{5}x + 2, \frac{1}{2}y + 4\right)$

$\rightarrow \left(\frac{1}{5}(5) + 2, \frac{1}{2}(4) + 4\right)$

$\Rightarrow (3, 6)$

$(3, 6)$

c) The point (m, n) exists on the graph of y = f(x). If the transformation y = 2f(2x) + 5 is applied to the graph, the transformed point is (4, 7). Find the values of m and n.

$(x, y) \rightarrow \left(\frac{1}{2}x, 2y + 5\right)$

$(8, 1)$

$\frac{1}{2}x = 4$ $\quad$ $2y + 5 = 7$

$x = 8$ $\quad$ $y = 1$