6.4: Sum, Difference, and Double-Angle Identities

Example 4: Determine the exact value for each expression.

(a) $\sin \frac{\pi}{12}$

$$\frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12}$$
$$= \frac{\pi}{4} - \frac{\pi}{6}$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$
$$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

(b) tan 105°

$$\tan 105^\circ = \tan(135^\circ - 30^\circ)$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\tan 135^\circ - \tan 30^\circ}{1 + \tan 135^\circ \tan 30^\circ}$$

$$\tan 135^\circ = -1$$
$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$= \frac{-1 - \frac{1}{\sqrt{3}}}{1 + (-1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\left(\frac{\sqrt{3}}{\sqrt{3}}\right) - 1 - \frac{1}{\sqrt{3}}}{\left(\frac{\sqrt{3}}{\sqrt{3}}\right) 1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{-\sqrt{3}-1}{\cancel{\sqrt{3}}} \cdot \frac{\cancel{\sqrt{3}}}{\sqrt{3}-1}$$

$$= \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

Another way--

$$\tan 105^\circ = \frac{\sin 105^\circ}{\cos 105^\circ} = \frac{\sin(60^\circ + 45^\circ)}{\cos(60^\circ + 45^\circ)}$$

$$= \frac{\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ}{\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ}$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$$

$$= \frac{\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{\cancel{4}} \cdot \frac{\cancel{4}}{\sqrt{2}-\sqrt{6}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}}$$

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8, 15(a), 16(b)